MODUL_2022

Machine Component Design and Mechanics

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Vector mechanics

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INTRODUCTION

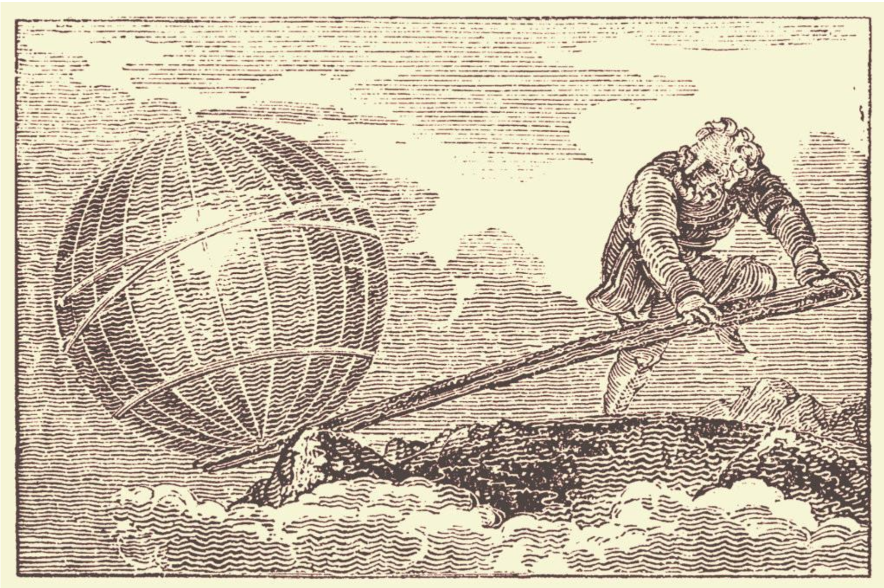
Vector mechanics serves as the fundamental framework for understanding and analysing the behaviour of physical systems, ranging from the motion of celestial bodies to the mechanics of intricate machinery. It provides a powerful toolset to describe and predict the motion and equilibrium of objects, making it a cornerstone of engineering, physics, and applied mathematics.

At its core, vector mechanics deals with the study of forces and their effects on objects. Unlike scalar quantities, which possess only magnitude, vectors possess both magnitude and direction. This critical distinction allows us to precisely characterize the motion and interactions of objects in three-dimensional space.

The principles of vector mechanics find their roots in Sir Isaac Newton's laws of motion. These laws, along with the concept of vector addition and subtraction, form the building blocks of the subject. By applying these principles, scientists and engineers can gain a deep understanding of the complex dynamics involved in various systems.

One of the fundamental concepts in vector mechanics is the concept of equilibrium. Equilibrium occurs when an object's net force and net torque become zero, resulting in a balanced state. Understanding equilibrium is essential for designing structures, analyzing the stability of objects, and predicting the behaviour of systems at rest. The first main chapter of this book is about this concept.

Vector mechanics also plays a crucial role in studying motion and its underlying causes. By employing vector principles, we can analyse the kinematics of objects, which involves studying the geometric aspects of motion without considering the forces that cause it. The application of vector calculus techniques, such as differentiation and integration, further enables us to delve into the dynamics of systems, accounting for the forces at play. This is the topic of the second main chapter.

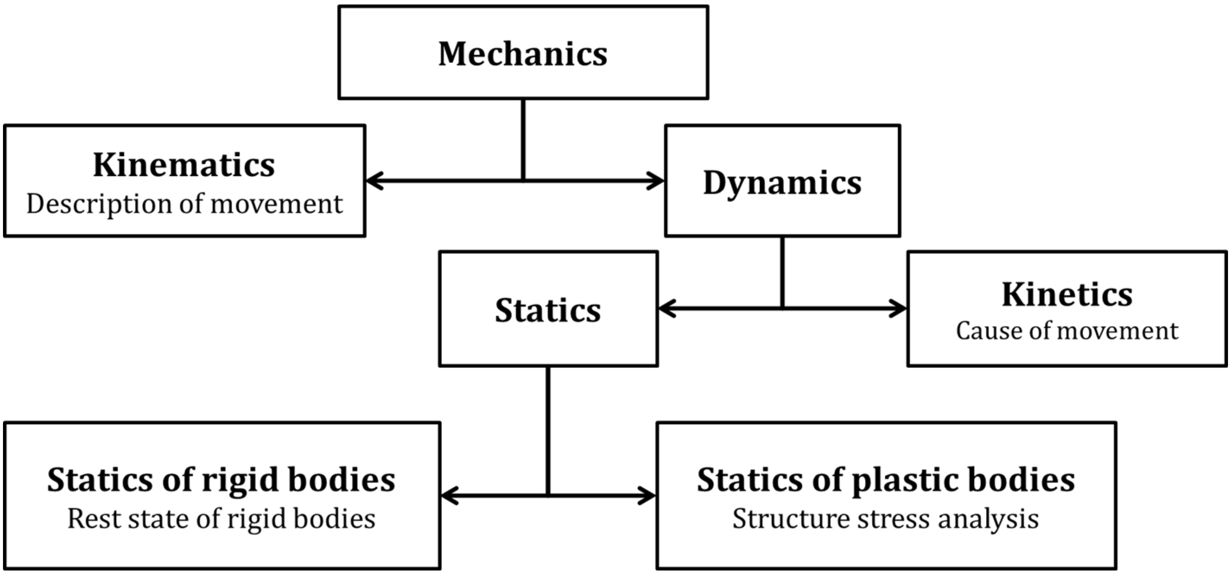


Mechanic's Magazine (cover of bound Volume II, Knight & Lacey, London, 1824

1 STATICS OF RIGID BODIES

1.1 ESSENTIAL BASIC TERMS AND CONCEPTS

The science of mechanics is a vast, diverse science. The most common categorization of mechanics is shown in the image below:



This textbook focuses on the mechanics of rigid bodies and structures. The first chapter is about statics of rigid bodies. The problems within this category are always calculated for a single moment in time, thus not taking transient effects into account.

This means that the bottom line of statics is that the inspected bodies and structures preserve their state of movement.

1.1.1 RIGID BODY

Definition: The distance of two randomly selected points of the body is constant.

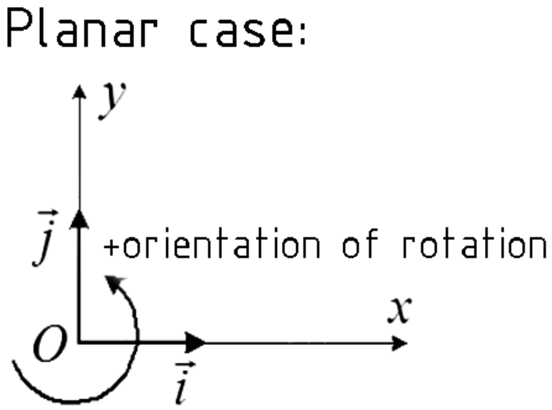
This means that any force magnitude can be applied to the rigid body, it will not change its shape. This book focuses on rigid bodies, which are clearly idealized models. In reality, solid bodies and structures behave differently, but rigid body mechanics is needed to establish a foundation of knowledge for more complex, less idealized problems.

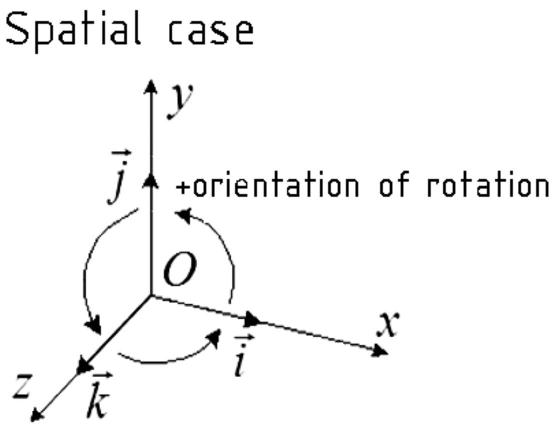
1.1.2 POINT IN MECHANICS

It can be considered as a rigid body with negligible dimensions.

1.1.3 REFERENCE SYSTEM

Right-handed cartesian coordinate system both in plane and in space.





where:

$\bar{i}$ - unit vector in the direction of x axis

$\bar{j}$ - unit vector in the direction of y axis

$\bar{k}$ - unit vector in the direction of z axis

1.1.4 THE FORCE

Definition: The mutual effect of two bodies on each other if it results in the change of movement state or deformation.

- Symbol: F, dimension: N (Newton)
- Occurs with or without contact

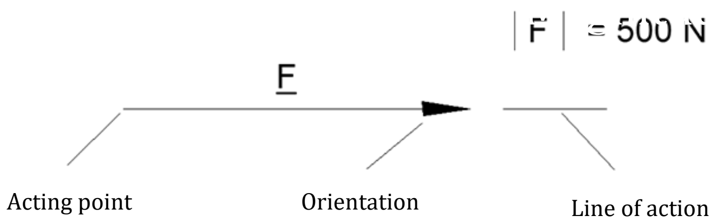
In case of contact, the force can be:

- Concentrated, acting at a single point
- Distributed on a line: p= G/l
- Distributed on a surface: p= F/A

Distributed forces can be substituted with a concentrated force acting in the centroid of the contact.

Eucleidan vector:

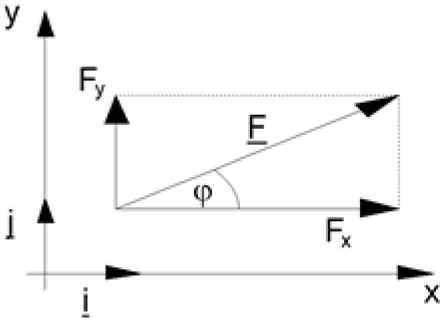
- Magnitude
- Direction
- Orientation



Vector operations are applicable.

Multiple forces can act on a body at the same time. This is called a force system. Based on **Newton's superposition principle**, the forces can be decomposed or summed in the direction of two (non-parallel) lines of action in a plane common to the force.

1.1.5 PLANE DECOMPOSITION OF THE FORCE:



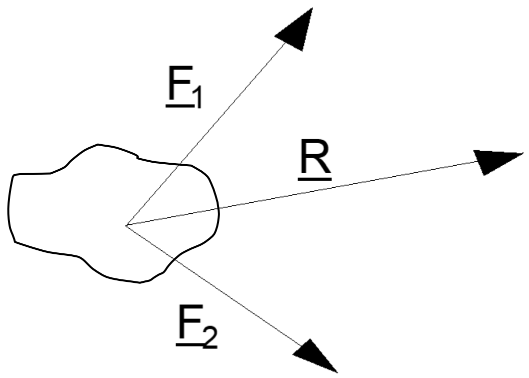
$F_x = F \cos \varphi$

$F_y = F \sin \varphi$

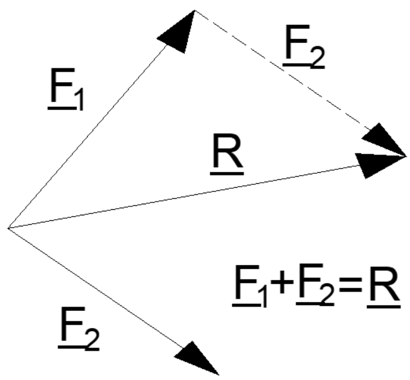
Vector equation: $\overline{F} = F_x\bar{i} + F_y\bar{j}$

Magnitude: $\overline{|F|} = F = \sqrt{{F_x}^2 + {F_y}^2}$

1.1.6 FORCE-ADDITION BY DRAFTING



Free body diagram
Length scale

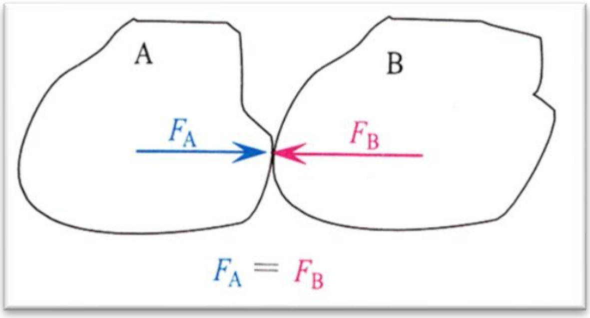


Vector diagram
Force scale

1.2 FUNDAMENTAL THEOREMS OF STATICS

1.2.1 THEOREM 1.: NEWTON'S III. AXIOM:

The principle of action-reaction, which states that the action of two bodies on each other is always equal to each other and has the opposite orientation.



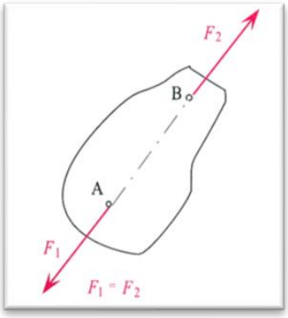
This has very important consequences:

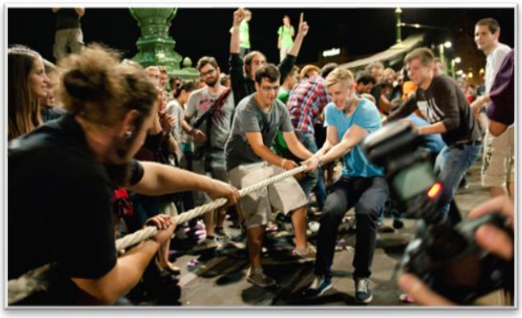
- **A force cannot exist alone, it can be exerted only if it has "something to hold onto".**
- **The direction of a force only depends on which body is in the selected reference frame.**

It is essential to understand this fundamental nature, to truly comprehend forces.

1.2.2 THEOREM 2.: THE REQUIREMENTS OF BALANCE

Two forces acting on a body are in balance only if the line of action of the two forces is common, their magnitude is the same and their orientation is opposite.

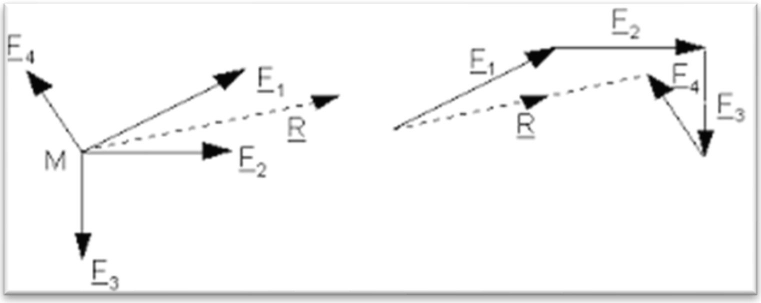


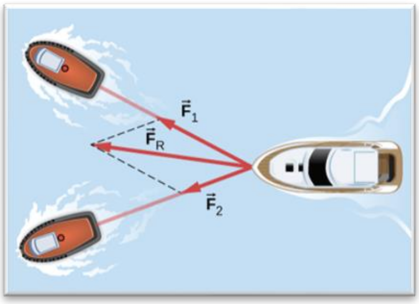


1.2.3 THEOREM 3.: RESULTANT FORCES

a force system with a common point of attack can always be replaced by a single, resultant force equivalent to the system. The acting point of the resulting force is the same as the acting point of the force system.

$$\bar{R} = \sum \bar{F}_i = \bar{F}_1 + \bar{F}_2 + \bar{F}_3$$

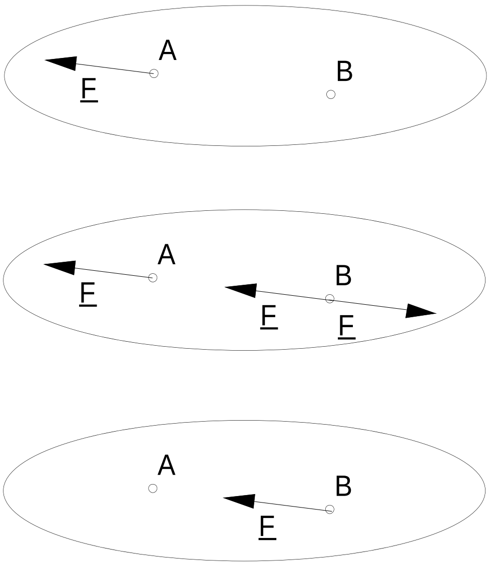




1.2.4 THEOREM 4.: SHIFTING OF FORCES

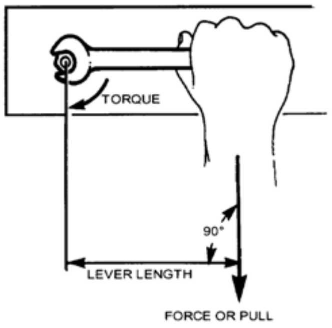
The effect of a force system acting on a rigid body does not change if we add or subtract another force system that is in balance on its own.

The consequence of the theorem is that forces acting on a rigid body can be shifted along their line of action.



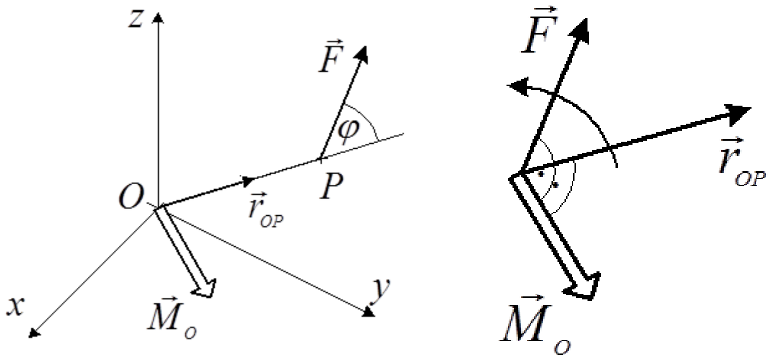
1.3 MOMENT

Definition: the rotational effect of a force around a point or axis.



Simplified formula:

$$M = F * k$$



Definition: $\overrightarrow{\boldsymbol{M}_{\mathbf{0}}} = \vec{\boldsymbol{r}}_{\boldsymbol{OP}} \times \vec{\boldsymbol{F}}$

Where: $\overrightarrow{M_O}$: moment in point O to the acting point of the force (P).

$\vec{r}_{OP}$: Distance vector from point O to the acting point of the force (P).

$\vec{F}$: The force vector.

The moment in a point is a vector.

Magnitude: $\left|\overrightarrow{\boldsymbol{M}_{\boldsymbol{O}}}\right| = \left|\overrightarrow{\boldsymbol{r}_{\boldsymbol{OP}}}\right|\left|\vec{\boldsymbol{F}}\right| \boldsymbol{sin\varphi}$

Orientation of the rotation comes from the right-hand rule:

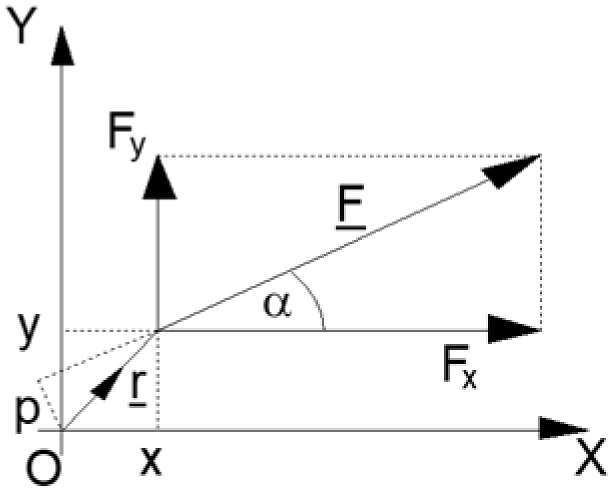
The moment vector is perpendicular to $\vec{r}_{OP}$ and $\vec{F}$.

$\vec{r}_{OP}$, $\vec{F}$ and $\overrightarrow{M_O}$ forms a right-handed system.

1.3.1 MOMENT THEOREM

The moment of a force at a point is equal to the sum of the moments of the components of the force at the same point.

In other words, the moment of the resultant of a force system at any point in the plane is equal to the sum of the moments of the forces at the same point.



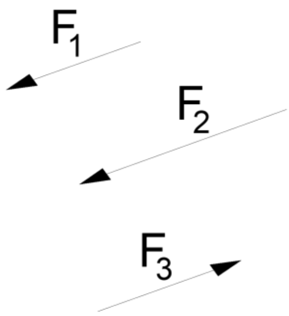
$$\underline{M} = \underline{r} \times \underline{F} = \begin{vmatrix} i & j & k \\ x & y & 0 \\ F_x & F_y & 0 \end{vmatrix} =$$

$$= (x \cdot F_y - y \cdot F_x)\underline{k}$$

1.3.2 RESULTANT OF PARALLEL FORCES

The resultant force can be determined from: $R = \sum F_i$, thus the magnitude, direction and orientation are known. However, the line of action is unknown! **The resultant replaces the force system**, so the moment of the resultant on any point is the same as the moment of the force system. The consequence of this is the moment theorem.

$$\sum F_i * k_i = R * k_0$$



Where: F_i: Forces of the force system,

k_i: Leverage of the forces,

R: The resultant force,

k_0: Leverage of the resultant

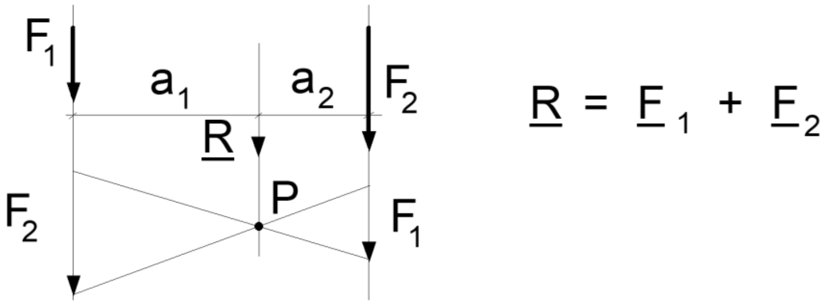
The moment of the force system is zero on the line of action of the resultant, so:

$$\sum F_i * x_i = 0$$

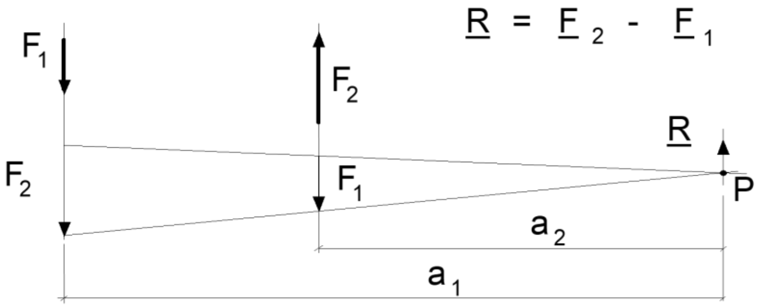
Where x_i is the distance between the lines of actions of each force and of the resultant.

Drafting method of finding the line of action of the resultant force:

Forces of same orientation:



Forces of opposite orientation:

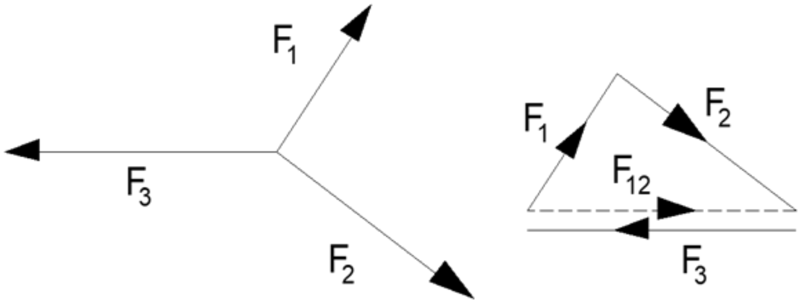


Simply draw the forces in a correct vector scale on each other's lines of action, then connect the ends of the forces with the points of action. The intersection of the connecting line will be on the line of action of the resultant force.

1.4 BALANCE OF THREE FORCES

Conditions:

1. Their lines of action intersect at one point.
2. Their vector triangles are closed and have a continuous arrow flow.



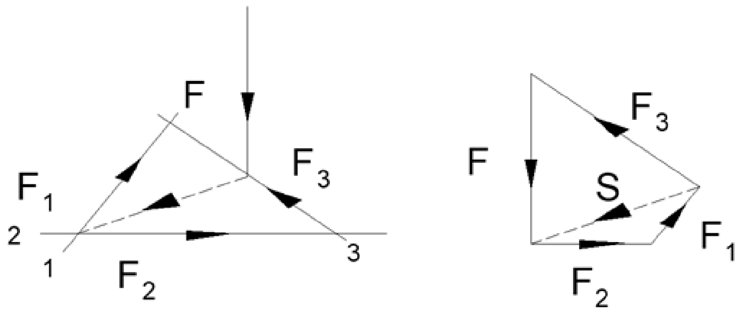
The balance of three forces can be simplified as the balance of two forces by substituting any two of the three forces with their resultant.

1.5 BALANCE OF FOUR FORCES

Conditions:

1. Their lines of action intersect at one point.
2. Their vector polygons are closed and have a continuous arrow flow.

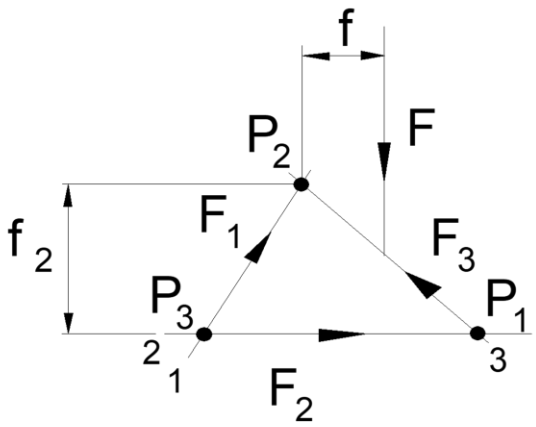
Cullmann's drafting method: $F + F_1 + F_2 + F_3 = 0; \quad \rightarrow \quad F + F_{12} + F_3 = 0; \quad F_{12} = S$



The base idea of Cullmann's drafting method is that the resultant of two forces will always go through the intersection of the lines of action of those forces. Thus, if we search the balance of four forces, two "pairs" or sub-resultants of forces can be created of which the intersection points can be found easily. These sub-resultants are only in balance if they fulfil the requirements of balance of two forces, being in line and equal in magnitude. Thus, the resultant force must contain the two intersection points we just found. These points determine the line of action of the resultant.

Ritter's calculation method:

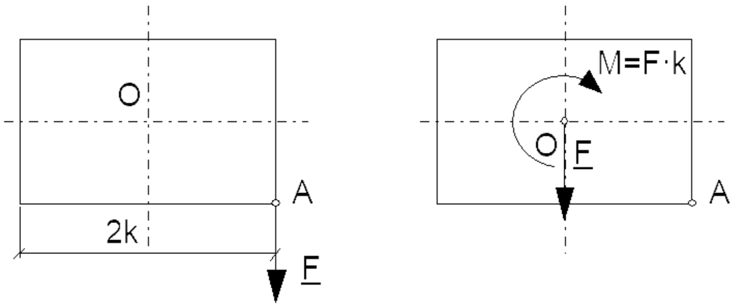
1. Select the intersection of the lines of action of the unknown forces in pairs.
2. The points thus obtained (P_1, P_2, P_3) are the main points of the respective forces. (E.g., the main point P_2 is the main point of the force F_2.)
3. Write the moment equilibrium equations for the main points. (E.g., determination of force F_2)



$$\sum M_{P2i} = -f * F + f_2 * F_2 = 0;$$

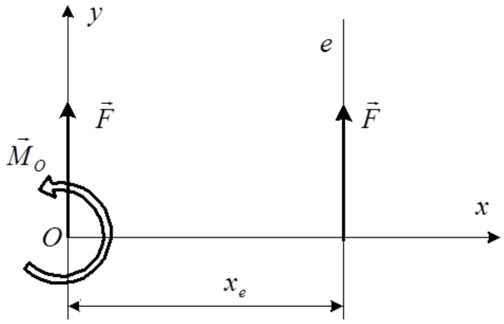
$$\xrightarrow{\text{thus}} F_2 = \frac{f}{f_2} * F;$$

1.6 REDUCTION OF FORCES



Reduction of a force is transferring the force F acting at point A to point O.

Result: the force F at point O and a moment M = F * k rotating around point O in the same direction as the original F force at point A.



The force system was replaced with the $\vec{F}$ and $\vec{M}_O$ reduced vector pair.

Defining the location of the acting line of the resultant:

$$M_O = x_e F; \rightarrow x_e = \frac{M_O}{F}$$

1.7 THE REDUCED VECTOR PAIR OF THE FORCE SYSTEM:

Theorem: The reduced vector pair is equal to the force system, thus replacing it.

The vector pair is a force and a moment vector.

The force vector is the vectoral sum of the forces in the system:

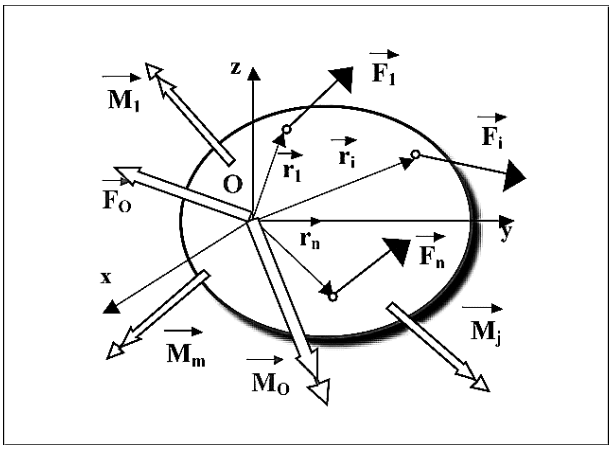
$$\vec{F} = \sum_{i=1}^{n} \vec{F}_i$$

The moment vector is the vectoral sum of the moment vectors in the system and the moments of the forces in the system calculated at the given point.

$$\vec{M}_A = \sum_{j=1}^{m} \vec{M}_j + \sum_{i=1}^{n} \vec{r}_{Ai} \times \vec{F}_i.$$

1.8 CLASSIFICATION OF FORCE SYSTEMS BASED ON THEIR REDUCED VECTOR PAIRS

Depending on the reduced vector pair, force systems can be categorized.



I. $F \times M = 0$;
1. $F=0$; $M \neq 0$; the resultant is only moment
2. $F \neq 0$; $M=0$; a single force through the point
3. $F \neq 0$; $M \neq 0$; a single force through the central line: $r = (F \times M)/(F^2)$
4. $F=0$; $M=0$; equilibrium

II. $F \times M \neq 0$;
The simplest form of the resultant:
A single force and a moment vector parallel with it.

1.9 EQUILIBRIUM EQUATIONS

F=0; M=0;

$\Sigma F_{xi} = 0$;

$\Sigma F_{yi} = 0$;

$\Sigma F_{zi} = 0$;

$\Sigma M_{xi} = 0$;

$\Sigma M_{yi} = 0$;

$\Sigma M_{zi} = 0$;

In statics, inspected systems sustain their state of movement. This is achievable only if the resultant force and moment vectors are 0, as acceleration of masses requires a force.

These equations are very important, they are the base of calculations in statics.

1.10 CONSTRAINTS

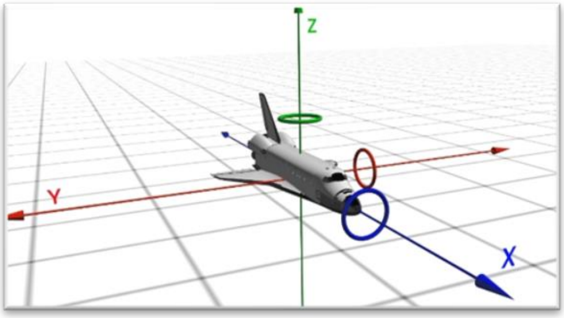
Constraints are movement-constraining elements, connections and forces that reduce the degrees of freedom of a body.

The number of independent coordinates that determine the current position of the body is called the **degree of freedom**.

Degrees of freedom of rigid bodies

in the case of spatial movement: 6

in the case of plane motion: 3



Degrees of freedom of material point

in the case of spatial movement: 3

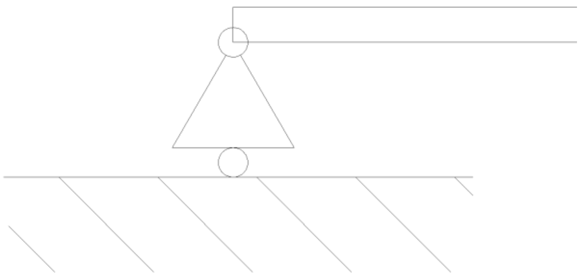
in the case of plane motion: 2

The forces acting on a body originating from constraints are called constrain forces or reaction forces. The forces loading the body are called active forces.

1.10.1 ROLLER (SLIDING)

The roller, or sliding constraint allows the constrained point to move freely in a plane but restricts the movement perpendicular to that plane. This kind of constraint allows rotations in every direction. Thus, the reaction force can be only perpendicular to the given plane, and the reaction moment is zero.

One unknown variable: the magnitude of the reaction force

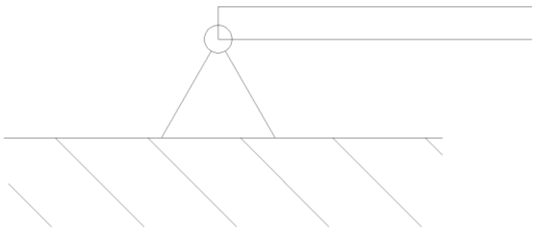


Symbol

1.10.2 FLAT JOINT (HINGED, PINNED)

The flat joint or hinged (pinned) constraint allows rotations in every direction but restricts every translation. Thus, the reaction force can act in any direction, but the reaction moment is zero.

Two unknown variables: the magnitude and direction of the reaction force



Symbol

1.10.3 CANTILEVER (FIXED)

The cantilever or fixed constraint restricts every rotation and translation in every direction. Thus, the reaction force can act in any direction, and the reaction moment is not zero.

Three unknown variables: the magnitude and direction of the reaction force and the reaction moment.

Symbol

1.10.4 A BEAM LOADED ONLY AT ITS ENDS

If the beam's mass is neglected and is only loaded by forces at its ends, then the line of action of the force in the beam is always in line with the beam.

One unknown variable: the magnitude of the normal force in the beam

Symbol

Trusses are built up of beams loaded only at its ends – or at least they are like that in an idealized model. The simplification that this kind of modelling gives, makes the calculation simpler by far, but only gives negligible errors in the calculated values.

1.10.5 ROPE

Similar to the beam loaded only at its ends, the force in a rope is always in line with the rope, or if the rope is curved, the acting line of the force in the rope is always tangent to the rope at the given point.

Differently from the beam, the force in a rope is always pulling, the rope is always in tension, compression is not possible.

One unknown variable: the magnitude of the pulling force in the rope

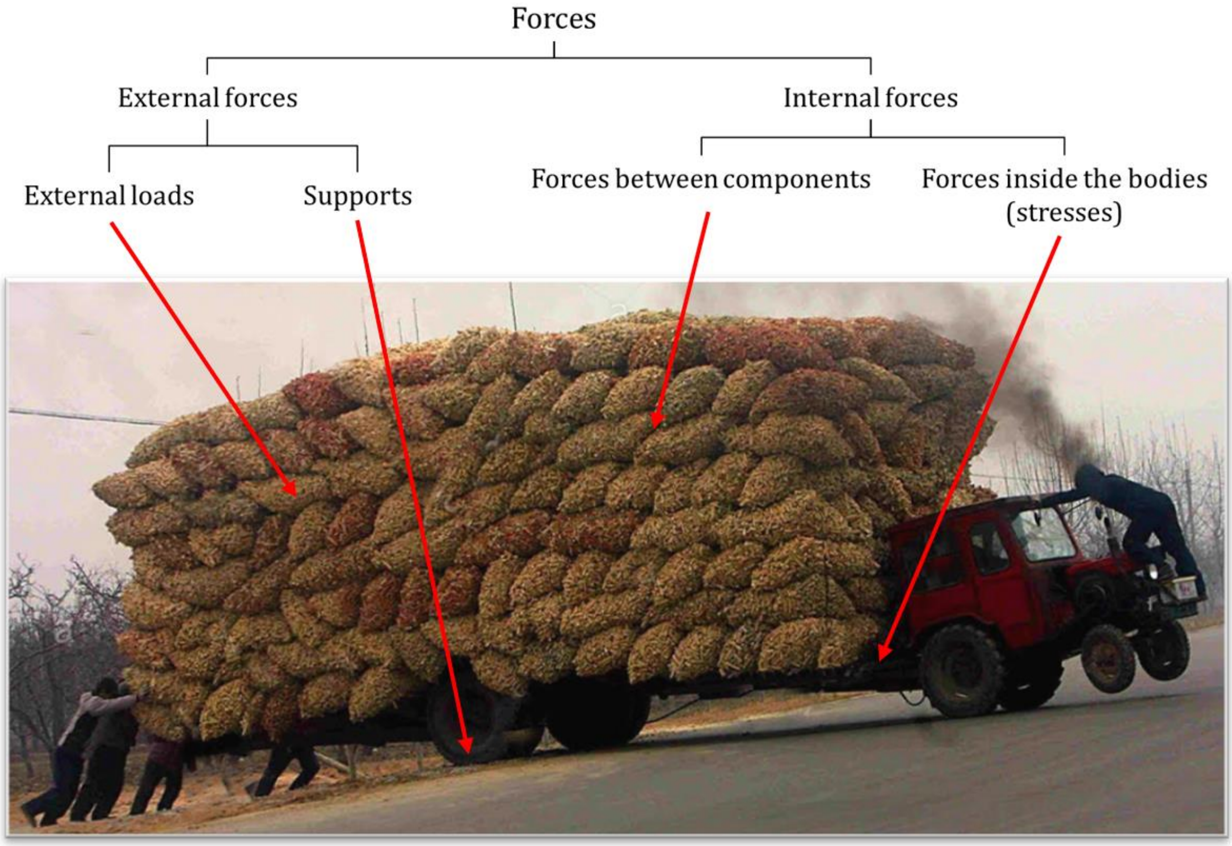
Symbol

1.11 INTERNAL FORCE SYSTEM OF RIGID BODIES

As previously discussed, the external forces acting on a system are the constraint (reaction) forces at the supports and constraints, and the loading (active) forces.

The external load forces exerted at one point of the structure has to “travel” somehow to the supports – that is the purpose of the structure or part in the first place. This “travel” of force means that in every cross-section of the part, an internal force system acts – burdening the structure with a certain intensity.

This internal, force intensity vector is also called “stress”. In this textbook, stresses are not discussed, but later, when studying the mechanics of solid bodies, stresses will be very important.

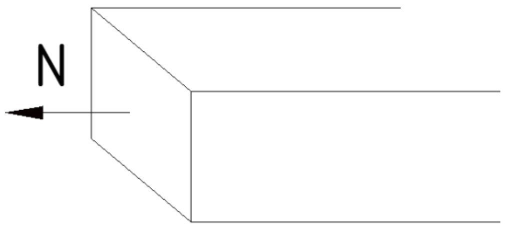


The load of a cross-section is defined by the reduction of the force system on one side of the cross-section to its centroid.

There are different types of internal forces depending on the types of loads.

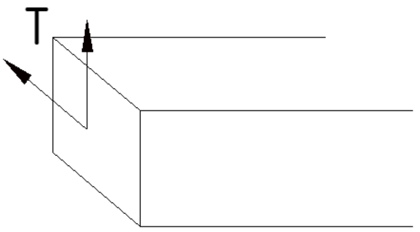
1.11.1 NORMAL LOAD

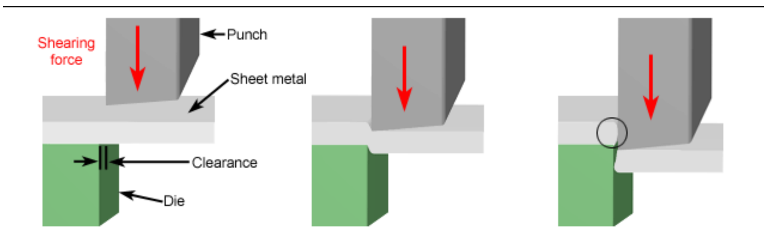
Normal load is when the force vector is perpendicular to the inspected cross-section. It can be tension (positive) or compression (negative).



1.11.2 SHEAR LOAD

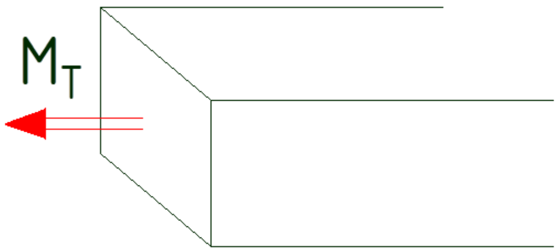
Shearing load is when the force vector is parallel with the inspected cross-section – thus perpendicular to the normal load.





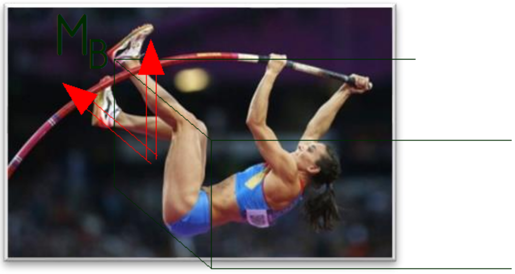
1.11.3 TORSION

Torsion load is when the moment vector is perpendicular to the inspected cross-section.



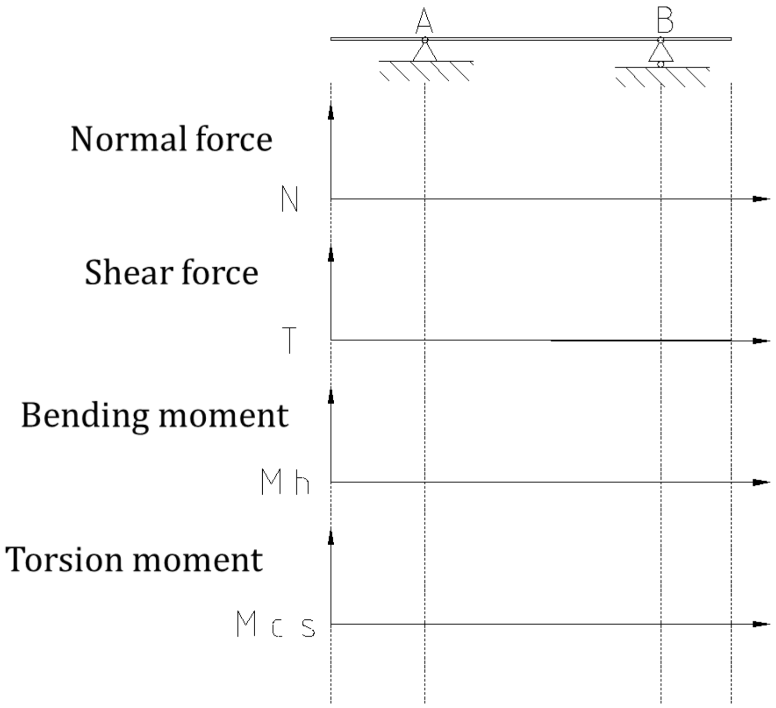
1.11.4 BENDING

Bending load is when the moment vector is parallel with the inspected cross-section – thus perpendicular to the torsion moment vector.



1.12 LOAD, SHEAR, AND MOMENT DIAGRAMS

Basic loads can be plotted along the beam like this:



Relation between load and internal forces:

$$\frac{dT}{dx} = p(x);$$

$$T(x) = T_0 + \int p(x)dx$$

Where T_0 is the initial value of shear force at x=0.

The internal shear force diagram therefore is the integral of the shear load diagram.

Respectively, the relation between the bending moment and shear force diagrams:

$$\frac{dM}{dx} = -T(x)$$

$$\mathrm{M}(x) = M_0 + \int -T(x)dx$$

Where M_0 is the initial value of bending moment at x=0.

Thus, the bending moment diagram is the integral of the shear force diagram.

1.13 RELATIONSHIP BETWEEN LOAD, SHEAR, AND MOMENT DIAGRAMS

The consequences of the differential relation between p(x), T(x) and M(x):

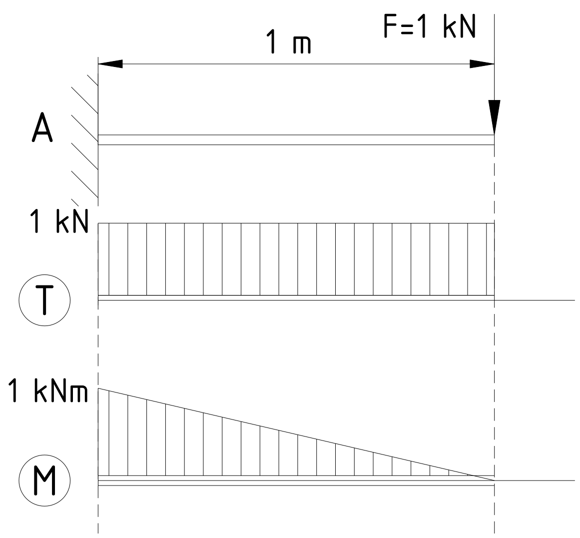
If the load (P(x)) is:	then the shear force (T(x)) is:	And the bending moment (M(x)) is:
0	0	Constant
Concentrated	Constant but not 0	Linear but not constant
Distributed, linear, constant	Linear but not constant	Second degree parabola
Distributed, linear, non-constant	Second degree parabola	Third degree parabola

$$P(x) = \frac{dT}{dx} = \frac{d^2M}{dx^2}$$

1.14 EXAMPLES FOR LOAD DIAGRAMS

The best way to understand the method of drawing load diagrams is through examples.

1.14.1 LOAD DIAGRAMS WITH A CONCENTRATED FORCE



The reaction force in point *A* is equal to F, because of the equilibrium equation in the vertical direction. This means that the shear force along the beam is 1 kN downwards everywhere.

The reaction moment at point *A* is F multiplied by the leverage, which is the length of the beam, 1 m. So, the moment M_A=1 kNm. Starting at this value, adding the integral of the shear force curve, we get the moment value at the end of the beam: M_1=M_A-1[kN]*1[m] = 0 kNm.

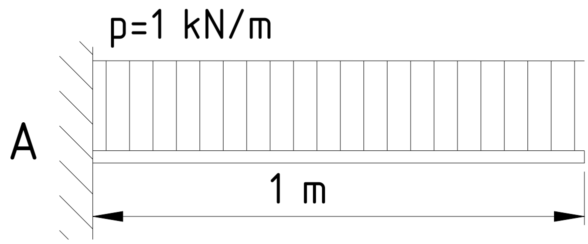
Connecting these points gives the bending moment curve.

1.14.2 LOAD DIAGRAMS WITH A DISTRIBUTED FORCE

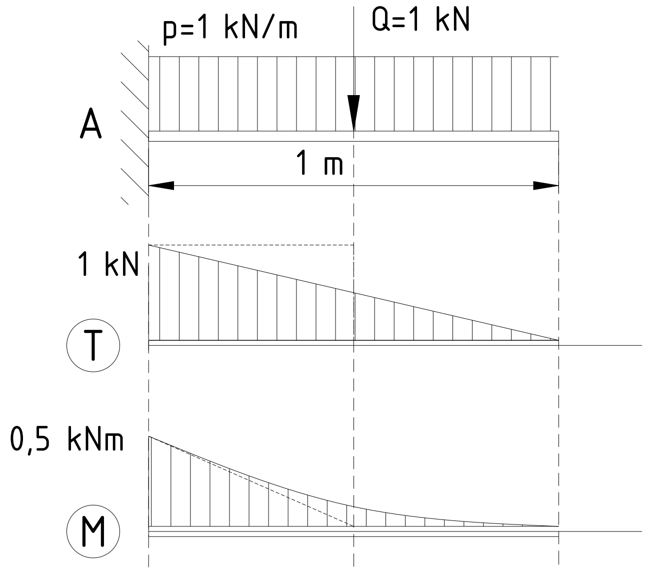
Distributed force means that the loading force does not act along a single acting line but is distributed in 1 to 3 dimensions:

- Distributed on a line (for example the own weight of a beam)
- Distributed on a surface (for example pressure in a container)
- Distributed on a volume (for example gravity or magnetic force fields)

In planar problems, only forces distributed on a line are discussed. The distributed force in this example is the following:



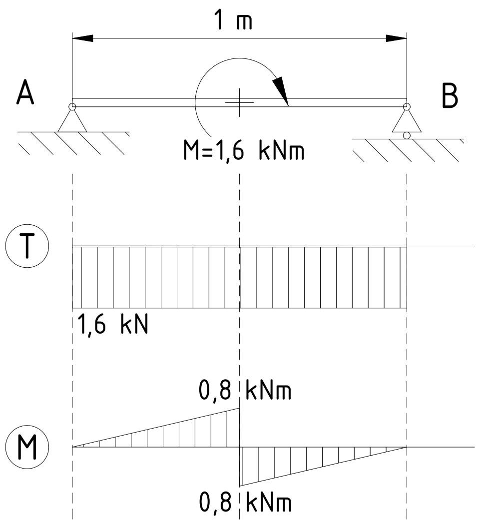
This means that the distributed force (p) is 1 kN for every m of the beam. As the beam is 1 m long, the total force can be calculated, which is 1 kN. The distributed force can be treated as a force system – thus it can be substituted with its resultant (Q):



The reaction force in point A is equal to Q, because of the equilibrium equation in the vertical direction. The shear force along the beam is decreasing from 1 kN. This can be drawn easily if first we draw the curve using Q, resulting in a curve with the dashed line in the image above. Then the end points of the curve must be connected giving the final shearing force curve. The reaction moment at point A is Q multiplied by the leverage, which is half the length of the beam, 0,5 m. So, the moment M_A=0,5 kNm. Starting at this value, the moment will gradually decrease, but parabolically.

If the distributed force is substituted with its resultant and draw the bending moment curve for Q, we get the tangent lines defining the real bending moment parabola.

1.14.3 LOAD DIAGRAMS WITH A CONCENTRATED MOMENT



The reaction force in point B is calculated by a moment equation on point A. like so:

$$M_A = 0 = 1 * F_B - 1{,}6$$

$$F_B = \frac{1{,}6}{1} = 1{,}6\ kN.$$

The reaction force in A is equal to the reaction force in B because of the equilibrium equation in the vertical direction. These values give the shear force diagram.

The reaction moment at point A and B is 0 because of the nature of the hinge and sliding constraints. If we start drawing the bending moment curve from point A, we start at 0, then add the integral value of the shear force curve up to the point where the concentrated moment is. Thus, we section the shear force curve in two parts, which is very important for getting a correct diagram. The bending moment value at this point:

$$M_K = 0 + 0{,}5 * F_A = 0{,}5 * 1{,}6 = 0{,}8\ kNm.$$

The concentrated moment does not appear on the shear force curve. Obviously, it should not appear as it is a moment, and the shear force curve represents forces. But the bending moment diagram represents moments, so now we have to take the concentrated moment into account. It will produce a sudden step in the curve.

Let's inspect the moment we calculated at this point. Its value is 0,8 kNm and the direction of rotation is counterclockwise (positive direction), as F_A points downwards. The concentrated moment is rotating clockwise (negative direction.) So, the moment after this point will be:

$$M_{K2} = 0{,}8 - 1{,}6 = -0{,}8\ kNm.$$

After this, we continue drawing the bending moment curve by adding the integral value of the shear force curve like so:

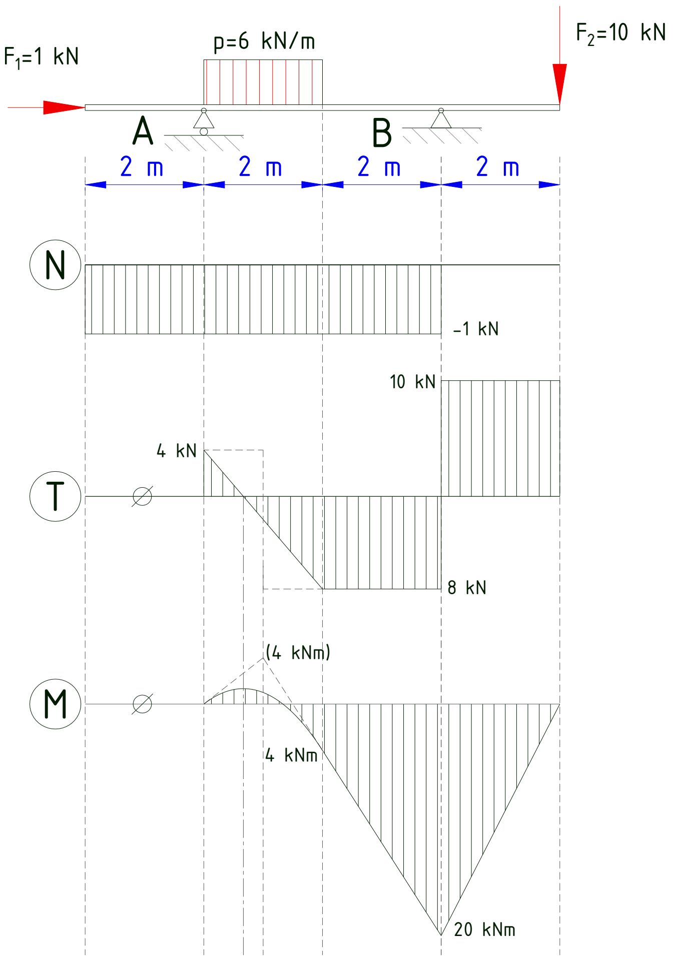
$$M_B = M_{K2} + 0{,}5 * F_A = -0{,}8 + 0{,}5 * 1{,}6 = 0\ kNm.$$

Thus, the bending moment in B is 0, as we expected.

Connecting these points gives the bending moment curve.

1.14.4 LOAD DIAGRAMS WITH COMPOSITE LOADS

Composite loads are when more than one type of load is exerted on the structure at once.



The horizontal component of the reaction force in point B is equal to the horizontal F_1 loading force because of the equilibrium equation in the horizontal direction. This value between the force and the hinge gives the normal force diagram.

The distributed load can be substituted by a concentrated force as shown earlier. Its value is:

$$Q = 2 * p = 2 * 6 = 12kN.$$

The vertical component of the reaction force in point B is calculated by a moment equation on point A. like so:

$$M_A = 0 = 4 * F_B - 1.Q - 6 * F_2$$

$$F_B = \frac{1 * Q + 6 * F_2}{4} = \frac{12 + 6 * 10}{4} = 18\ kN.$$

The reaction force in A is calculated using the equilibrium equation in the vertical direction:

$$\sum F_{yi} = 0 = F_B + F_A - Q - F_2$$

$$F_A = Q + F_2 - F_B = 12 + 10 - 18 = 4\ kN.$$

These values give the shear force diagram.

The reaction moment at point A and B is 0 because of the nature of the hinge and sliding constraints. The free ends of the beam can rotate freely, thus the bending moments here are also 0. Please note that only the reaction moment is 0 at points A and B, the internal moment in the beam can have a value other than 0.

The differential relation between the shear force and bending moment curves mean that the shear force curve shows the change of the bending moment curve.

Since the shear force is 0 between the left end of the beam and point A, the bending moment curve is not changing either, so the value is 0 in this region.

If we continue the bending moment curve from point A, we start at 0. Then, we should notice that the next section is the distributed force, so the bending moment curve will be a parabola. So, we should draw its tangent lines as discussed earlier:

$$M_1 = -4 * 1 = -4\ kNm;$$

$$M_2 = M_1 + 8 * 1 = -4 + 8 = 4\ kNm;$$

Now we can draw the parabola on the lines connecting these points. Please note, that the calculated M_1 is not the real bending moment value, the curve does not take this value, it is only part of the tangent lines.

Now we can continue drawing the curve after calculating the missing bending moment values:

$$M_3 = M_2 + 8 * 2 = 4 + 16 = 20\ kNm;$$

$$M_4 = M_3 - 10 * 2 = 20 - 20 = 0\ kNm;$$

Thus, the bending moment at the end of the beam is 0, as we expected.

Connecting these points gives the bending moment curve.

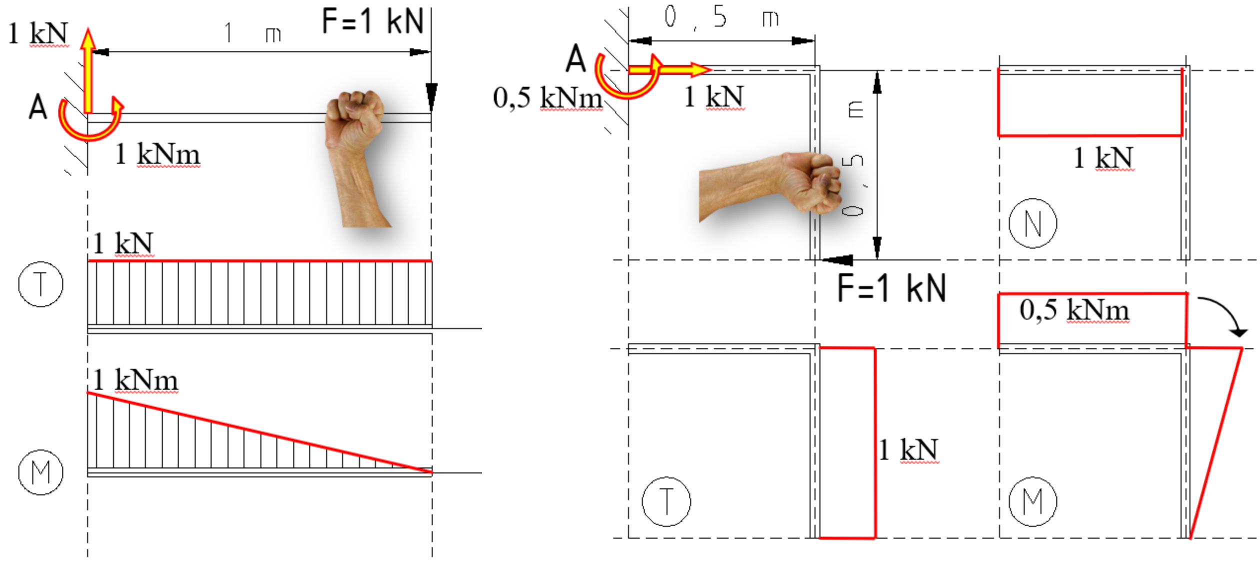
1.15 LOAD DIAGRAMS OF COMPOUND BEAMS

During engineering practice, compound beams are used quite often. These are not linear but have corners and branches.

The calculation and drawing method of shear force and bending moment curves is the same as in the case of straight beams. The most important difference is that the calculation of the resultant forces needs to use the leverages of the forces on the compound beam correctly. When drawing the curves, the reference frame always must be linked to the straight sections of the compound beam.

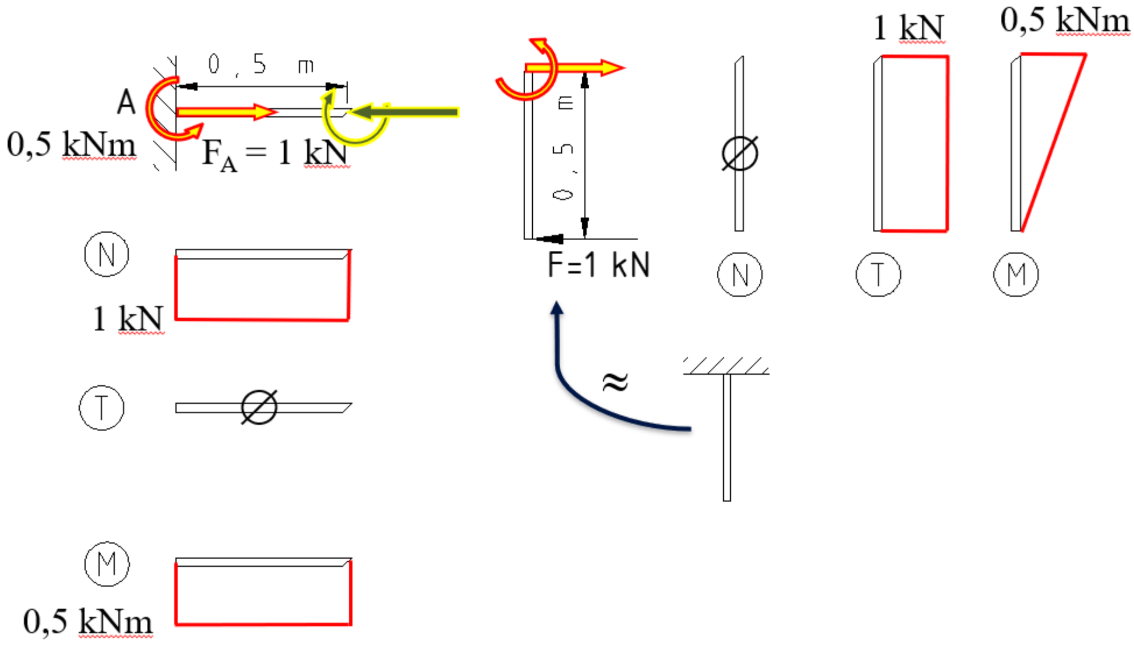
Consider this problem as multiple straight-beam problems joined together in certain angles. Please note, that at the corners and bifurcations, the loaded beams have an effect on the supported parts.

For example:

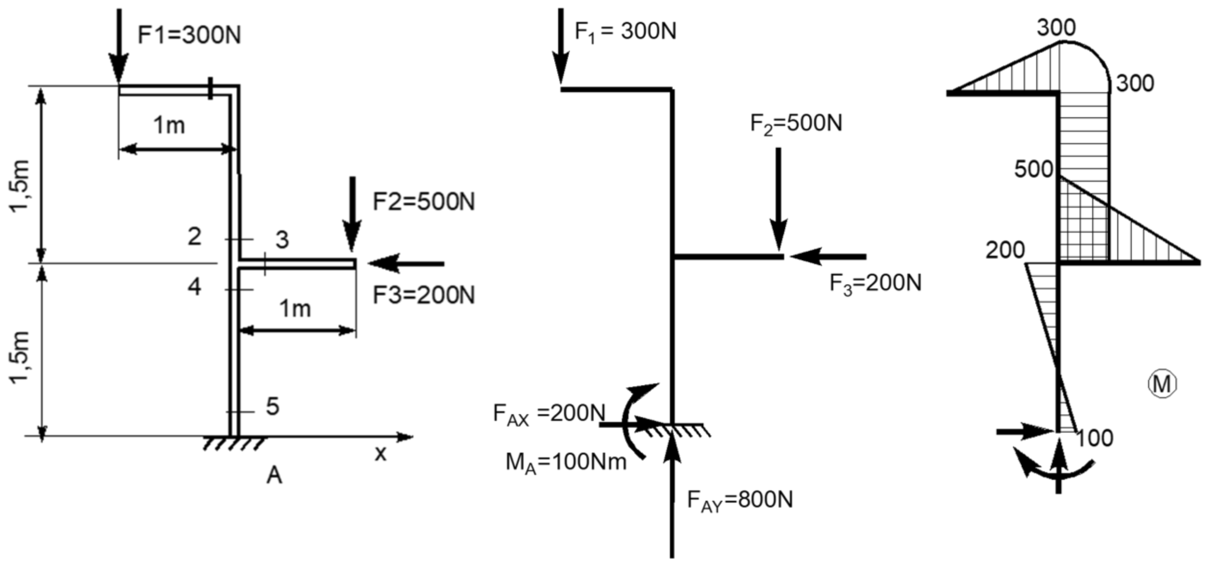


Here the beam is changed, which results in a different leverage of the force and different reactions.

If the straight segments of the beam are separated, the problem can be solved as two separate straight beam problems the same way discussed earlier:

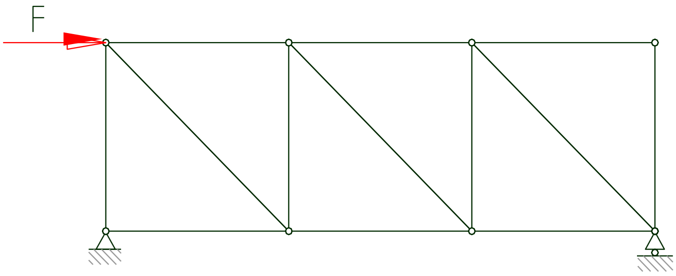


Another example for compound beam problems:



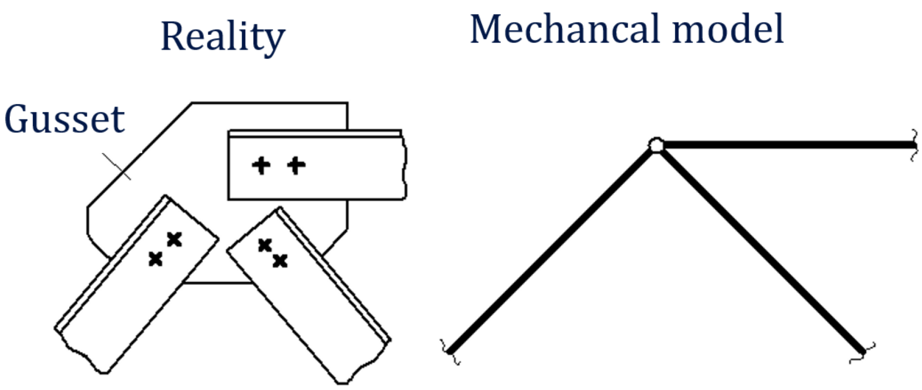
1.16 TRUSSES

1.16.1 PLANAR TRUSSES

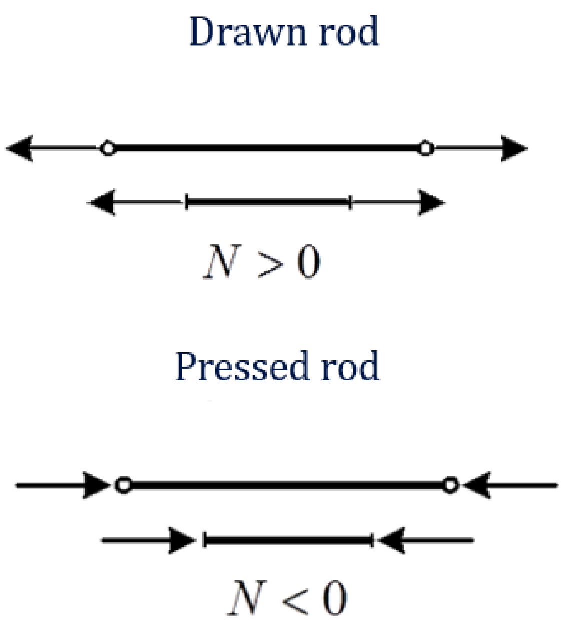


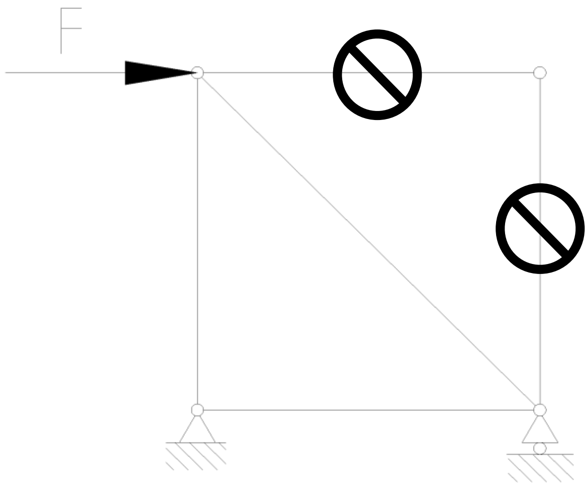
- Consists of straight rods
- The rods are connected by joints
- Loaded or supported on the joints only (only at the ends of the rods, never in between!)
- External statical determination: $n_b + n_k = s$
- Internal statical determination: $r = 2 * c - 3$

  Where r: the number of rods and c: the number of joints
- The weight of the structure is neglected



Because load is applied only at the nodes, only rod-direction forces act in the bars. The unloaded rods are called blind rods.





Blind rods

1.16.2 THE METHOD OF JOINTS

This truss problem solving method is described through an example.

Given:

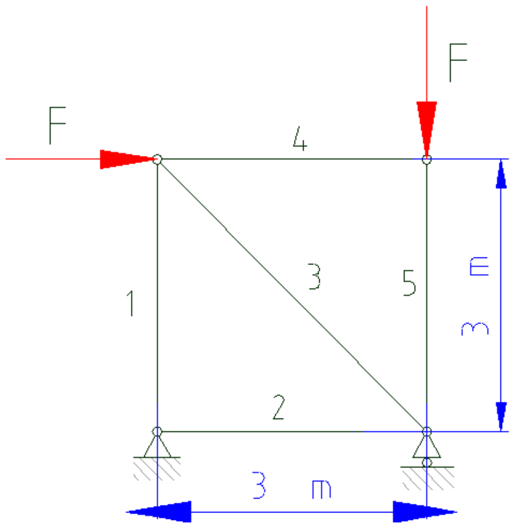
Dimensions and load of the structure

Task:

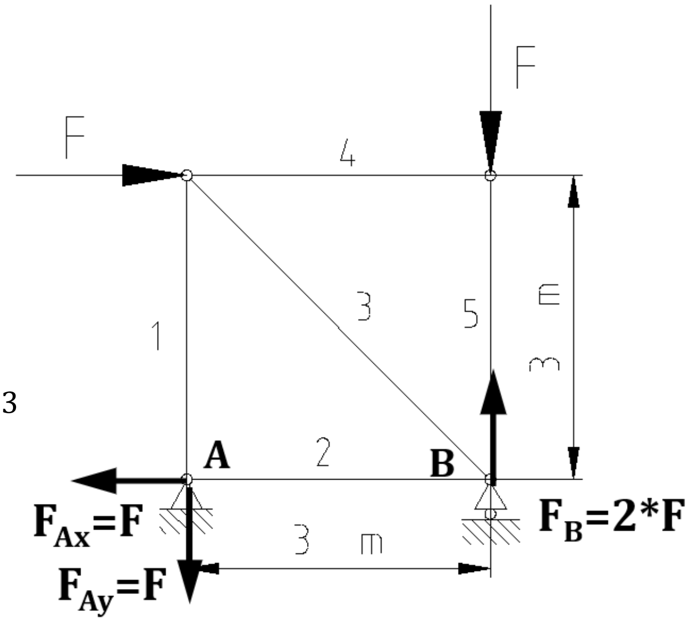
1. Determination of support forces
2. Determination of bar forces

Principle:

The structure as **a whole and in parts** is in balance!



Step 1. Determination of support forces



1.1. Balance of forces in x direction:

$F - F_{Ax} = 0 \rightarrow F_{Ax} = F \leftarrow$

1.2. Moment equation on point A:

$$\sum M_{Ai} = 0 = F_B * 3 - F * 3 - F * 3$$

$F_B = 2*F \uparrow$

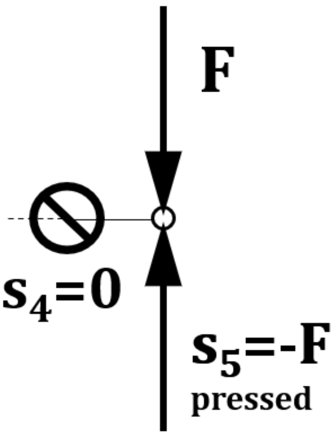
1.3. Balance of forces in y direction:

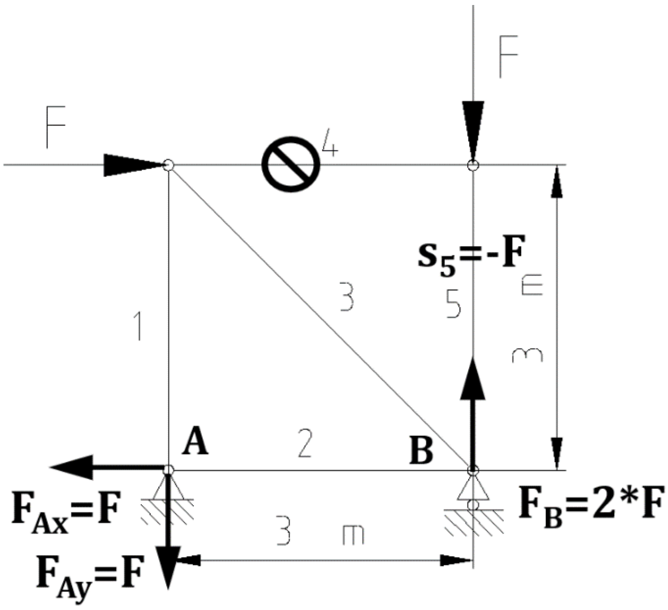
$F_B - F - F_{Ay} = 0 \rightarrow F_{Ay} = F \downarrow$

Step 2. Determination of bar forces

The structure as **a whole and in parts** is in balance! Thus, individual nodes are in balance. The task is simply finding the balance of intersecting forces at the nodes.

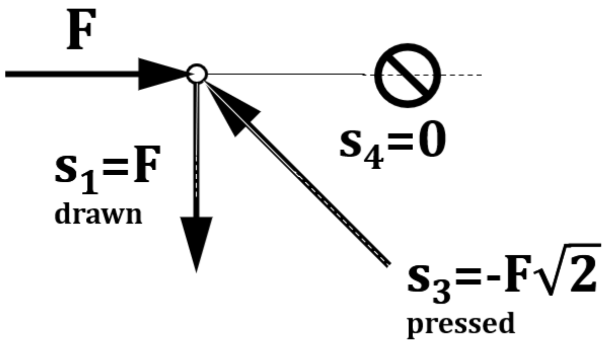
2.1. Upper right corner

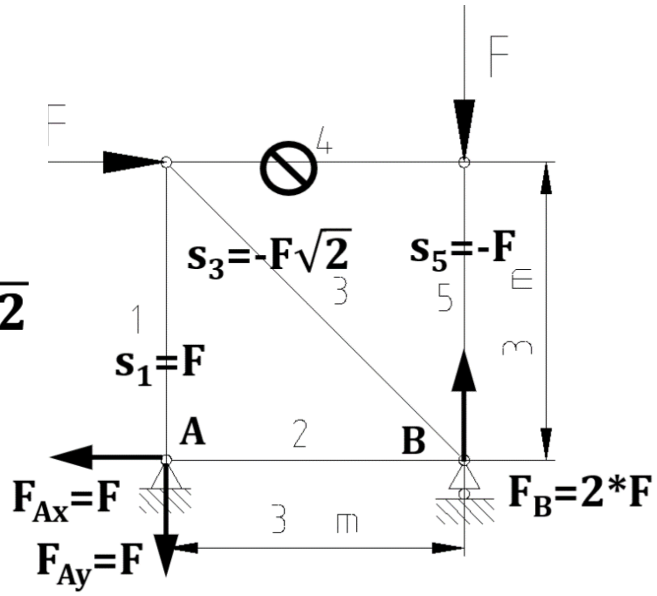




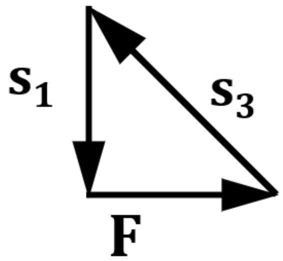
Always choose nodes with the fewest unknown variables!

2.2. Upper left corner

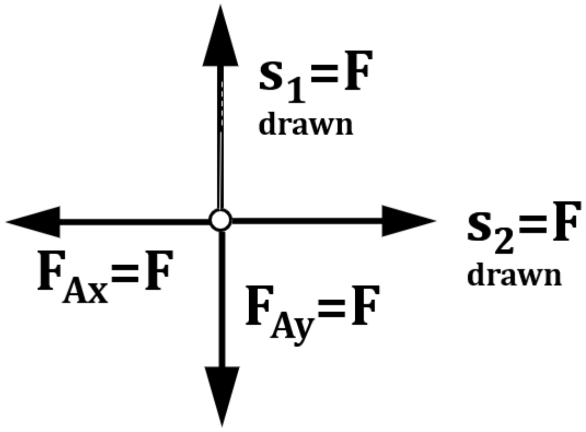


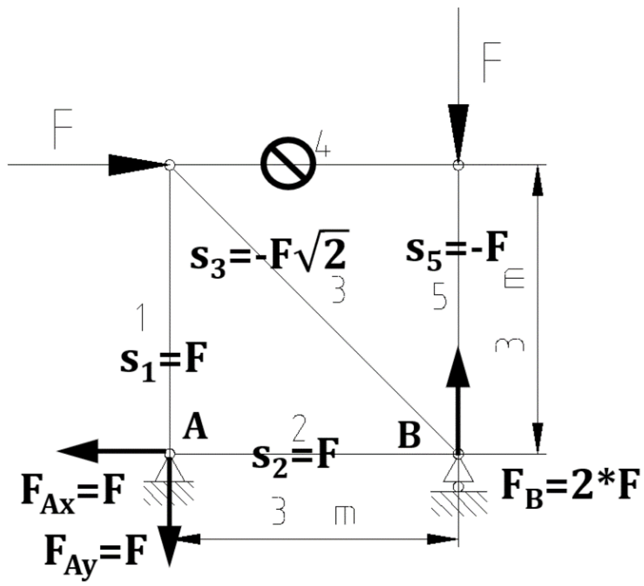


Vector diagram:



2.3. Lower left corner





Thus, the solution:

Support forces:	Bar forces:
F_{Ax}=F;	s_1 = F;
F_{Ay}=F;	s_2 = F;
F_B=2F;	$s_3 = -F\sqrt{2}$;
	s_4 = 0;
	s_5 = -F;

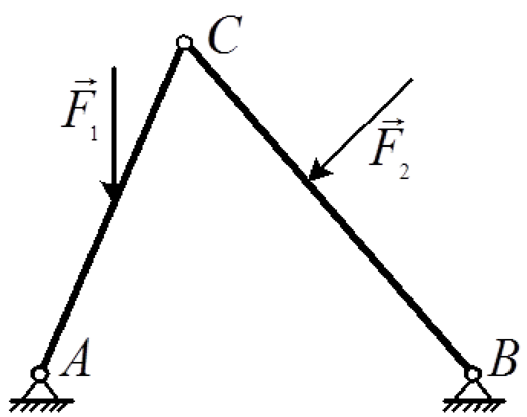
1.17 FRAMES

Three-hinge frames: structures that are internally not stabile, they would collapse without supports.

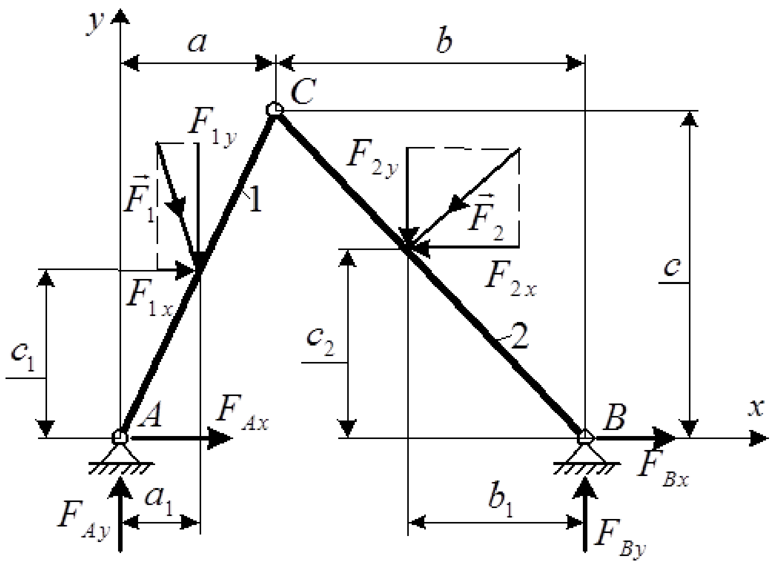
Example:

Given: Dimensions and load of the structure

Task: Determination of support forces



Forces in y direction can be determined by examining the whole structure. But not in x direction!

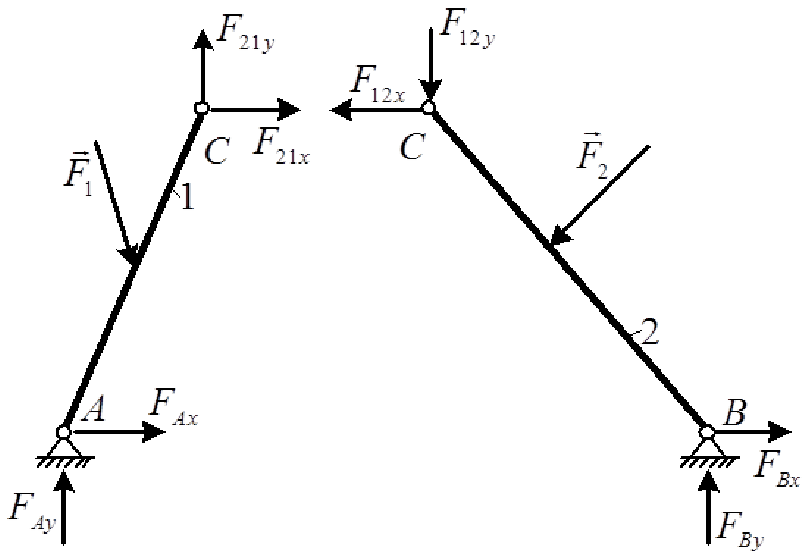


Forces in y direction are calculated using moment equations like so:

$$M_a = 0 = -c_1 F_{1x} - a_1 F_{1y} + c_2 F_{2x} - \left[a + (b - b_1)\right] F_{2y} + (a + b) F_{By}, \quad F_{By} = \dots \quad .$$

$$M_b = 0 = -c_1 F_{1x} + c_2 F_{2x} + \left[b + (a - a_1)\right] F_{1y} + b_1 F_{2y} - (a + b) F_{Ay}, \quad F_{Ay} = \dots \quad .$$

To calculate reaction forces in the x direction, the structure is divided in thought.



Horizontal reaction force components calculated using a moment equation at C only considering the left side of the structure:

$$M_c = 0 = -aF_{Ay} + cF_{Ax} + (c - c_1)F_{1x} + (a - a_1)F_{1y}, \quad \vec{F}_A \quad ,$$

$$F_x = 0 = F_{Ax} + F_{1x} + F_{21x} \quad \Rightarrow \quad F_{21x} = \dots ,$$

$$F_y = 0 = F_{Ay} - F_{1y} + F_{21y} \quad \Rightarrow \quad F_{21y} = \dots .$$

The same calculation carried out on the right side of the structure:

$$M_c = 0 = cF_{Bx} + bF_{By} - (c - c_2)F_{2x} - (b - b_1)F_{2y},$$

$$F_{Bx} = \dots \quad ,$$

$$F_x = 0 = F_{Bx} - F_{2x} - F_{12x} \quad \Rightarrow \quad F_{12x} = \dots \ ,$$

$$F_y = 0 = F_{By} - F_{2y} - F_{12y} \quad \Rightarrow \quad F_{12y} = \dots \ .$$

After calculating one of the horizontal reaction forces, the other can be easily calculated using a simple force equilibrium equation in x direction for the whole structure.

1.18 FRICTION

Friction is a resistive force acting between two contacting surfaces opposite of the direction of forces parallel to the contact surfaces. Friction always acts in a way to prevent movement.

1.18.1 THE COMPONENTS OF FRICTION

- Adhesion

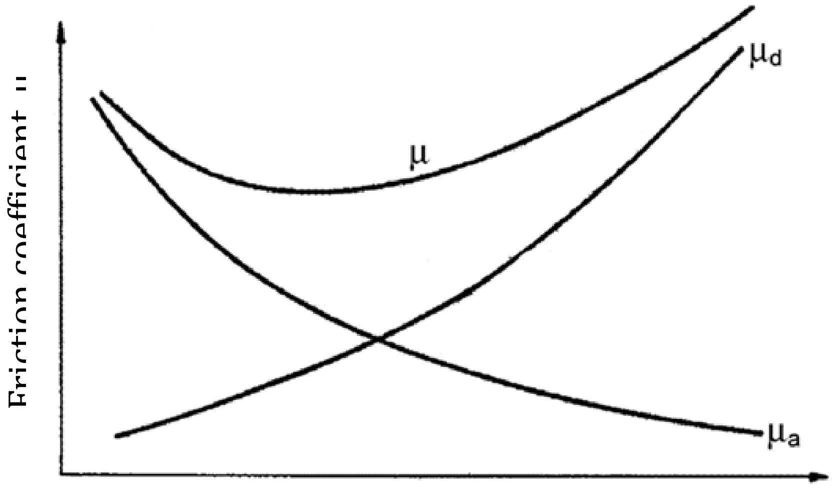
  If the atoms of the two contacting surfaces can get close enough to each other, a weak attractive force acts between them, thus producing a resistance when the surfaces try to move along each other. This component of friction is emphasized in the case of relatively smooth surfaces.

- Deformation

  In case of higher surface roughness, when two surfaces move along each other, small bits of pieces break down or are deformed on them. This deformation energy is lost in a form of resistance against movement.

A good example to demonstrate low and high roughness friction: pole vs. rope climbing.

These two components are always present in different ratios, depending on the roughness of **both** contacting surfaces:

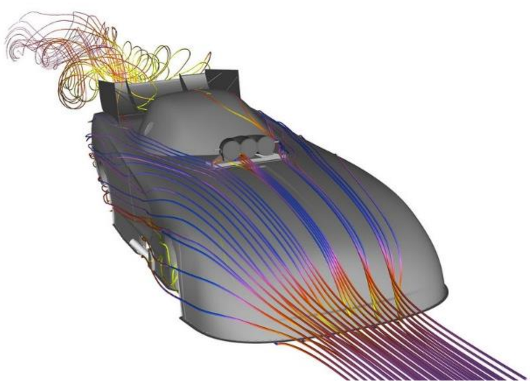


Averge surface roughness, R_a

1.18.2 IS THE FRICTION GOOD OR BAD?

In some cases, friction is harmful, or is meant to be reduced:

- Wear, abrasion
- Losses, resistances

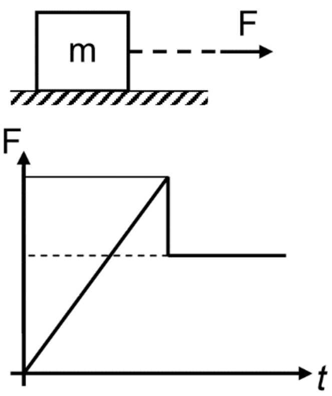
In engineering, when designing structures with moving parts, friction is like an arch enemy, as it greatly determines the efficiency of the designed machinery.

Many times, friction is useful or even necessary:

- Walking
- Breaking, starting
- Everyday life…

Just imagine what would happen without friction. Every unfixed object and creature would end up in lowest points of Earth's surface unable to move whatsoever.

1.18.3 FRICTION FORCE:



When a force parallel to the contacting surfaces (pulling force from now on) is applied to a body, the friction force acts in the contact point.

If the pulling force increases, the friction force increases as well – up to a maximum value. This is the static friction force.

Then when the body is moving, the friction force resisting the movement is lower than the static friction force. This is called the dynamic friction force.

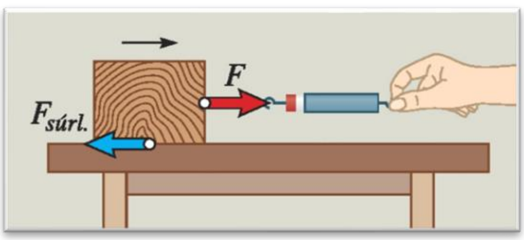
As this paragraph is about statics, we only discuss the case of static friction and especially the boundary condition of movement.

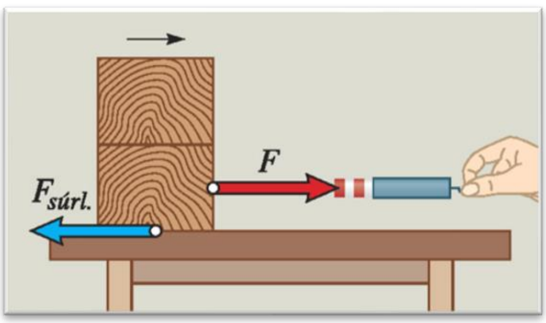
Friction force depends on:

- Roughness of both surfaces
- The magnitude of the forces compressing the surfaces together

It does NOT depend on:

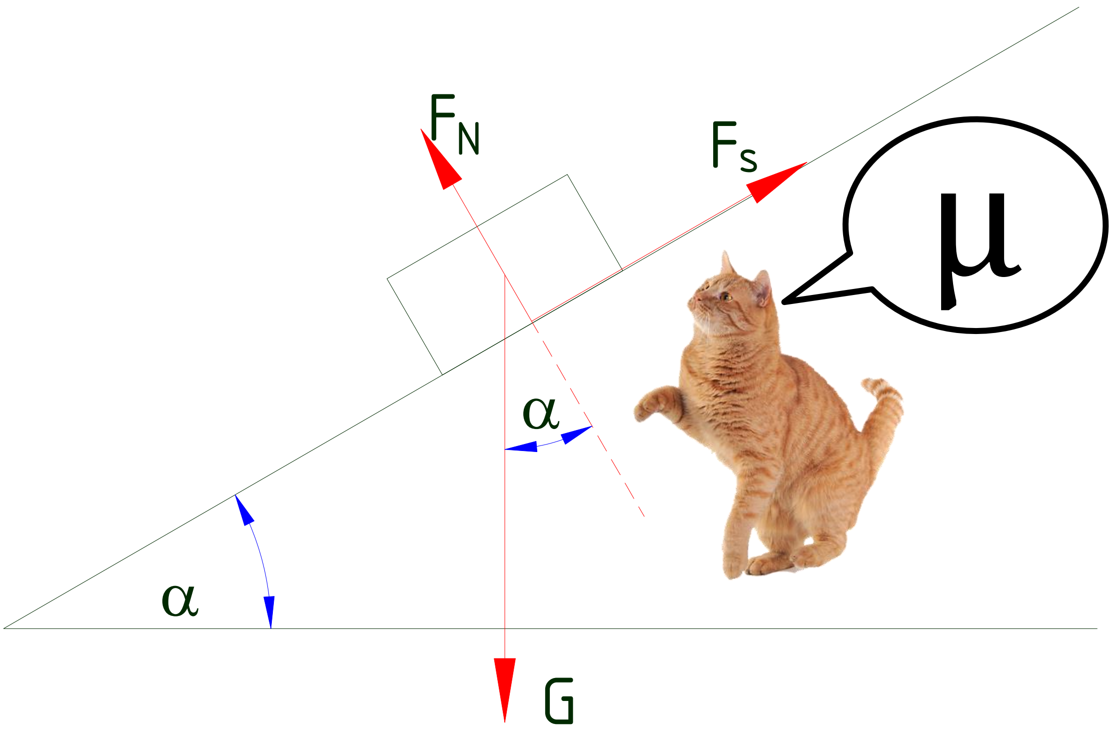
- The size of the surfaces





1.18.4 THE FRICTION COEFFICIENT

Definition: μ: The ratio between the normal and the maximum friction force is called the friction coefficient. It depends on the physical parameters of the contacting surfaces. It is a dimensionless value, as it is the ratio of two forces.



Thus,

$$\mathbf{F_S = \mu * F_N}$$

But only in the boundary state of sliding!

Where:

F_S: friction force

F_N: normal force compressing the surfaces together

μ: friction coefficient

1.18.5 THE FRICTION CONE AND THE BOUNDARY POSITION OF SLIDING

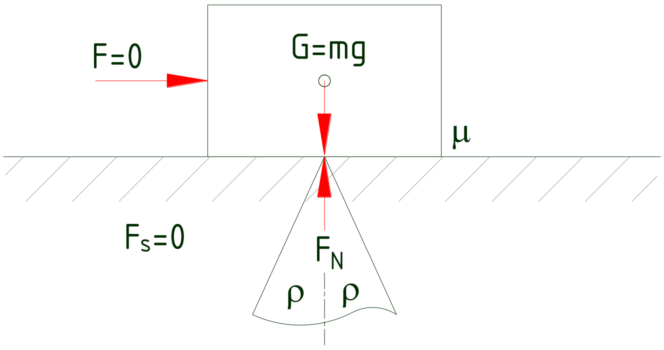
Definition: The friction cone is an imaginary cone, surface of which contains the boundary lines of action of the resultant forces of the compressing and the friction forces.

The friction cone has a half-angle. This angle is the angle of the resultant of the compressing and the friction forces (F_E), thus it is defined by them, like so:

$$\mathbf{tg}\,\rho = \frac{F_S}{F_N} = \mu$$

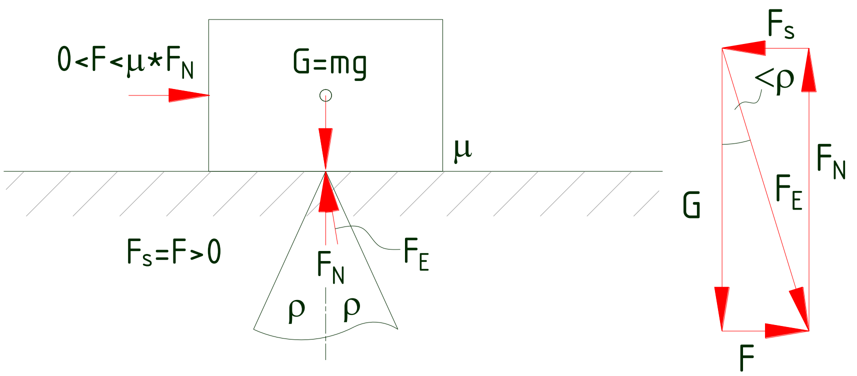
1.18.6 EXAMPLES FOR SPECIFIC CASES OF FRICTION

1. **The pulling force is 0.**



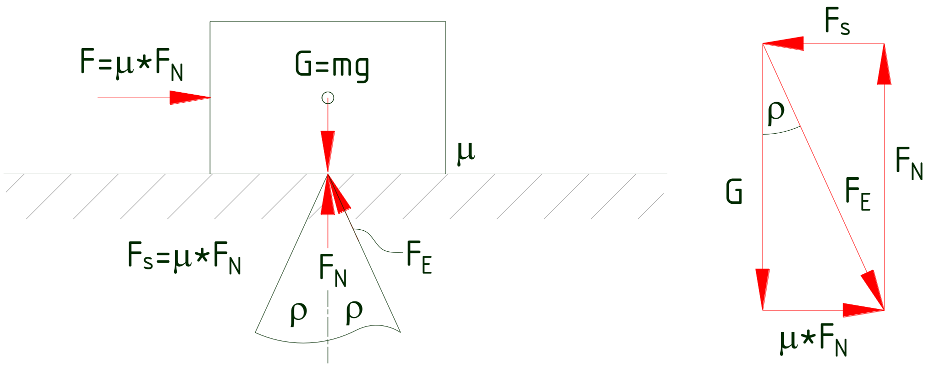
In this case, there is no point to draw a vector diagram, as all the horizontal forces are 0.

2. **The pulling force is greater than 0, but lower than the maximum value of the friction force**



On the vector diagram, the angle of F_E is demonstrated which is lower than **ρ**. This means that the object is stationary, and the friction force is equal to the pulling force.

3. The pulling force is equal to the maximum value of the friction force



On the vector diagram, the angle of F_E is equal to **ρ**. This means that the object is on the boundary state of movement, and the pulling force is equal to the maximum value of the friction force.

Until the instant when sliding occurs, the magnitude of the friction force is just enough to prevent movement.

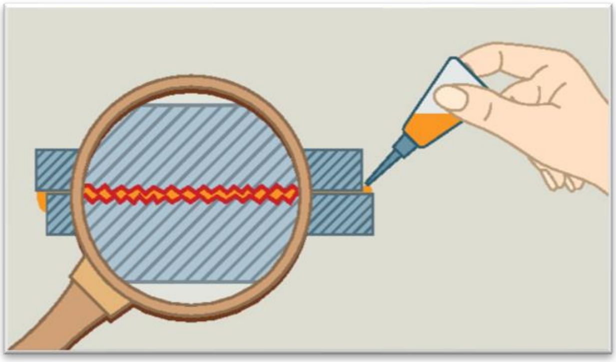
1.18.7 REDUCTION OF FRICTION FORCE

In engineering, friction needs to be reduced very frequently. There are different methods to do so.

- **Appropriate roughness:** Choosing both the surfaces roughness in a way that minimizes friction.

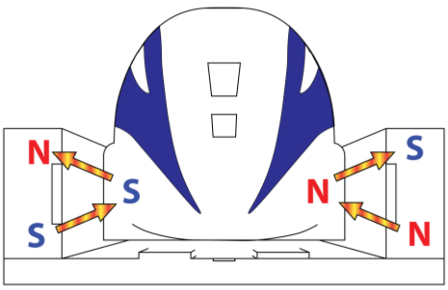
- **Bushing, coating:** Placing a third material between the two contacting surfaces that can slide more easily or is cheap to be replaced when worn.

- **Lubricant:** Similar to bushing and coating, but this time the third material is not quite solid. Oils and greases are widely used as lubricants, their purpose is to keep the contacting surfaces away from each other, preventing contact.



- **Rolling (wheels, bearings):** Eliminating sliding entirely, wheels are used to roll on one of the surfaces, carrying the other object, or enable rotation of a shaft within a bore.


- **Magnetic levitation:** Keeping the surfaces apart using magnets results in changing the friction of the surfaces to only air resistance, which is a lot lower in most cases.



1.19 ROLLING RESISTANCE

An obround object, a wheel for example, rolling on a surface without sliding is called clean rolling**.** This means that the contact point of the wheel is stationary relative to the surface it is rolling on.

However, the contact between the wheel and the surface is not a single point – it can not be, as it would produce infinite pressure. (See: $p = \frac{F}{A}$)

The contact surface is produced by two main factors:

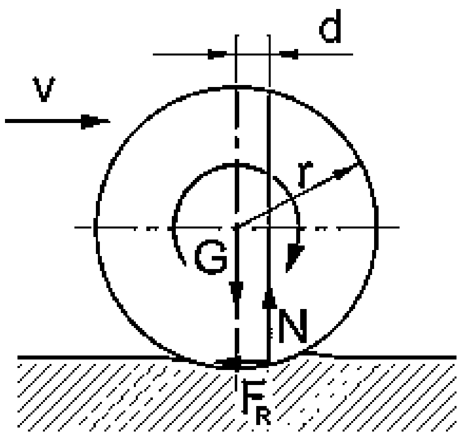
- Compression of the wheel
- Indentation of the ground

The consequence of this phenomenon is that the rotating wheel has a resistive moment produced by the supporting force distributed on the contacting surface.

This resistive moment is called rolling resistance and is calculated from the supporting force and the leverage (d) between the acting lines of the resultant of the supporting (distributed) force (N) and the force loading the wheels (G).

In the boundary state of rolling:

$$\boldsymbol{F_N * d = M_h}$$

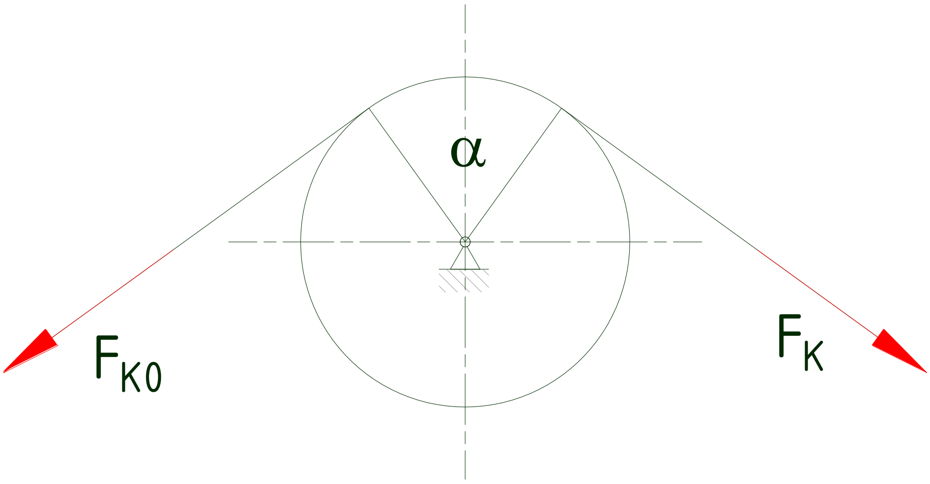


A very important requirement of clean rolling is a necessary friction between the wheel and the surface to prevent sliding.

1.20 ROPE AND BELT FRICTION

Winding up a rope on a bollard does not let it slip down. This phenomenon is caused by friction between the rope and the bollard.

Every rope or belt that is pushed on a cylindrical surface resists movement just like in standard cases of friction discussed earlier. The rope/belt friction depends on the contacting surfaces, the pulling forces in the rope and the central angle subtended by the rope on the surface. Please note, that this angle can be greater than 360°, in fact can be of any value.



Rope friction force:

$$\boldsymbol{F_K = F_{K0} * e^{\mu_0 * \alpha}}$$

Where:

μ_0: the coefficient of friction between the rope and the cylinder

α: central angle of the contacting rope segment [rad].

It is demonstrated, that increasing the central angle increases the rope friction exponentially.

2 KINEMATICS AND KINETICS

Kinematics deals with the description motion of bodies without examining the cause of the origin of the motion. The simplest moving body is a particle. A particle (point mass) defined as a body whose dimensions are negligible. If the physical element is not assumed to be particle, it is referred to as body. All bodies considered in this text are assumed to be rigid.

Two general types of particle motion may be identified. The first type is rectilinear translation, and this type of motion is straight line motion. The second general type of particle motion is called curvilinear translation.

2.1 KINEMATICS OF PARTICLE

We know the motion if we know the particle location of the particle with respect to a fix reference point.

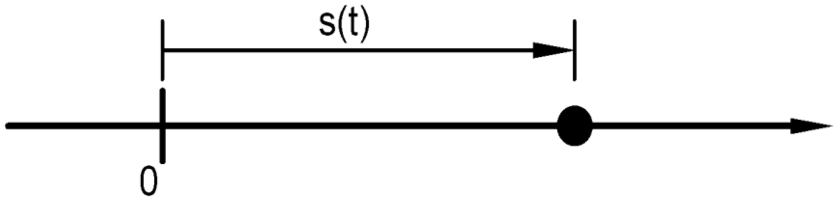


Figure 2.1.1 Rectilinear translation

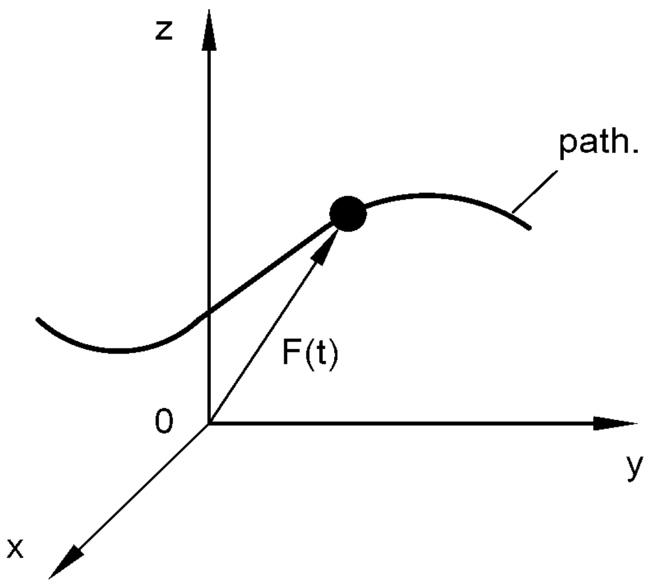


Figure 2.1.2 Curvilinear translation

In most general case a three-dimensional coordinate system is used to define position of particle. The position vector of particle is a vector drawn from reference frame to the particle.

$$\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$$

The trajectory of a particle is a vector function of time $\bar{r}$ (t), which defines the curve traced by moving particle, given by

$$\bar{r}(t) = x(t)\bar{i} + y(t)\bar{j} + z(t)\bar{k}$$

2.1.1 VELOCITY AND SPEED

The particle moves distance *Δs* during the time internal *Δt.*

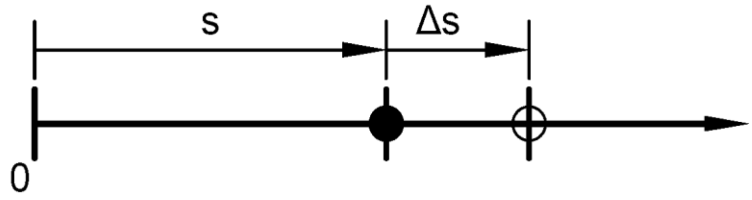


Figure 2.1.3

The average velocity v_{avg} is defined to be

$$v_{avg} = \frac{\Delta s}{\Delta t}$$

If $\Delta\bar{r}$ is change in the position vector in the time *Δt.*

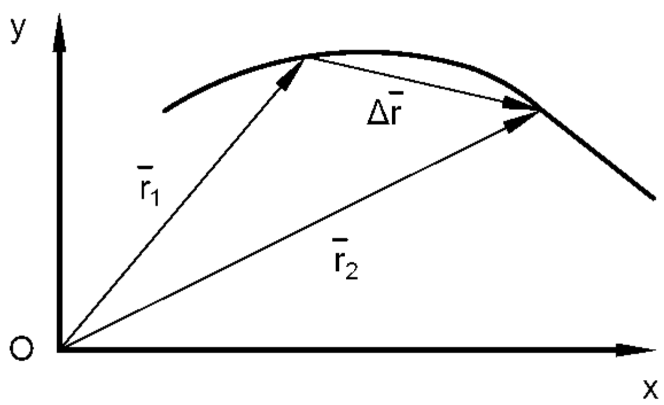


Figure 2.1.4

$$\overline{v}_{avg} = \frac{\Delta\overline{r}}{\Delta t} = \frac{\overline{r_2} - \overline{r_1}}{t_2 - t_1} \quad ms^{-1}$$

In the limit that time interval Δt approaches zero, the average velocity approaches the instantaneous velocity

$$\overline{v} = \lim_{\Delta t \to 0} \frac{\Delta\overline{r}}{\Delta t} = \frac{d\overline{r}}{dt} = \dot{x}\,\overline{i} + \dot{y}\overline{j} + \dot{z}\overline{k}$$

where: e.g.

$$\dot{x} = \frac{dx}{dt}$$

The speed of an object is magnitude of its velocity

$$v = \overline{|v|} = \frac{ds}{dt}$$

2.1.2 ACCELERATION

Acceleration is the rate at which velocity changes.

The average acceleration is defined by

$$\overline{a}_{avg} = \frac{\Delta\overline{v}}{\Delta t} = \frac{\overline{v_2} - \overline{v_1}}{t_2 - t_1}$$

The acceleration of the particle is the limit of the average acceleration as the time approaches zero, which is the time derivate

$$\overline{a} = \lim_{\Delta t \to 0} \frac{\Delta\overline{v}}{\Delta t} = \frac{d\overline{v}}{dt} = \dot{v} = \dot{v_x}\overline{i} + \dot{v_y}\overline{j} + \dot{v_z}\overline{k}$$

or

$$\overline{a} = \ddot{r} = \ddot{x}\,\overline{i} + \ddot{y}\overline{j} + \ddot{z}\overline{k}$$

2.1.3 RECTILINEAR MOTION WITH CONSTANT ACCELERATION

Considering any kind of movement due to constant acceleration it is important to find the position and velocity of the object at later time.

It gives the acceleration *(a)*, the initial position *(x_0)* the initial velocity *(v_0)*

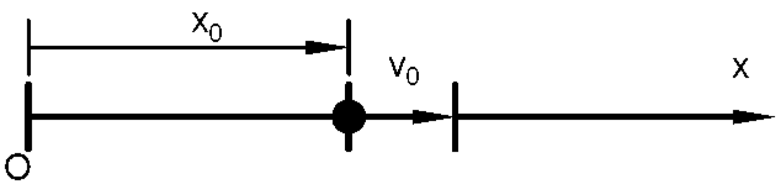


Figure 2.1.5

From definition average acceleration

$$a = \frac{\Delta v}{\Delta t} = \frac{v(t) - v_0}{\Delta t}$$

therefore

$$v(t) = v_0 + a\Delta t$$

From the definition average velocity, the displacement

$$\Delta s = v_{avg}\ \Delta t$$

$$\Delta s = \frac{v_0 + v(t)}{2}\Delta t = v_0\Delta t + \frac{a}{2}\ \Delta t^2$$

$$x = x_0 + v_0\Delta t + \frac{a}{2}\ \Delta t^2$$

The three equations are below

$$x = x_0 + v_0 t + \frac{1}{2}\ at^2$$

$$v = v_0 + at$$

a=constant

If x_0, v_0, and and a assumed to be positive, the displacement, velocity and acceleration diagrams would have the forms shown in Fig. 2.1.6.

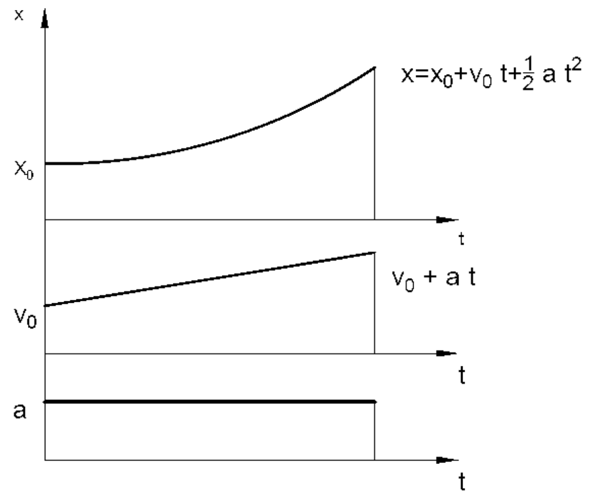


Figure 2.1.6

Example

A car starts from zero velocity and moves along a straight horizontal roadway.

The velocity diagram is redrawn in Fig. 2.1.7.

a. Using graphical methods, construct the displacement and acceleration diagrams
b. How far has the car travelled by the time it comes to rest?
c. Find the average velocity of the vehicle over the entire period of motion

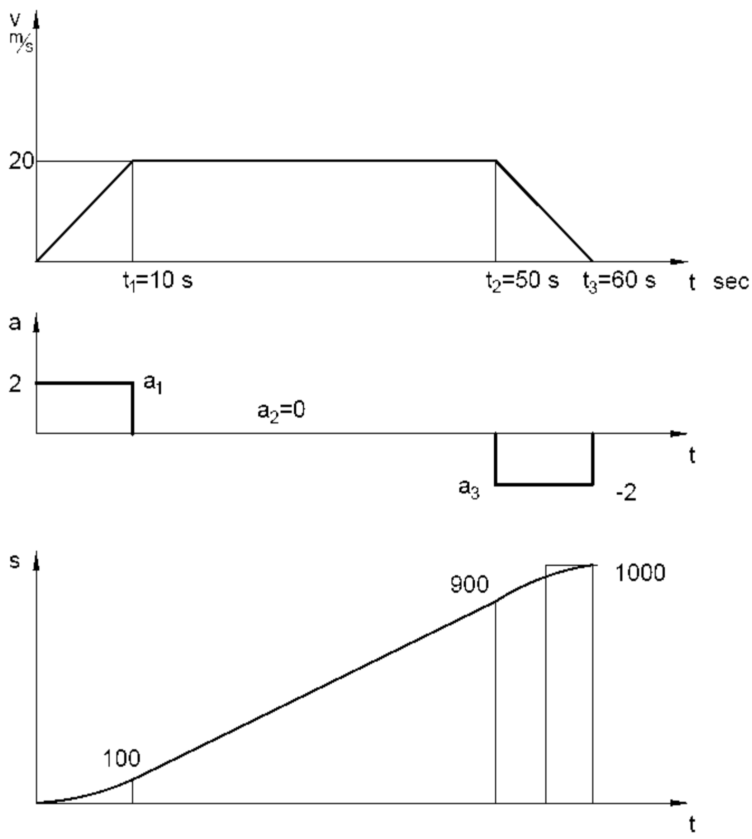


Figure 2.1.7

$$a_1 = \frac{v_1}{t_1} = \frac{20}{10} = 2\ ms^{-2}$$

$$a_3 = \frac{0 - v_2}{t_3 - t_2} = \frac{-20}{10} = -2\ ms^{-2}$$

$$s = \frac{1}{2} a_1 t_1^2 + v_1(t_2 - t_1) + v_2(t_3 - t_2) + \frac{1}{2} a_3 (t_3 - t_2)^2 =$$

$$= 100 + 800 + 200 - 100 = 1000\ m$$

$$v_{avg} = \frac{1000}{60} = 16.67\ m/s$$

2.1.4 CIRCULAR MOTION

Uniform circular motion is a motion in a circular path with constant speed.

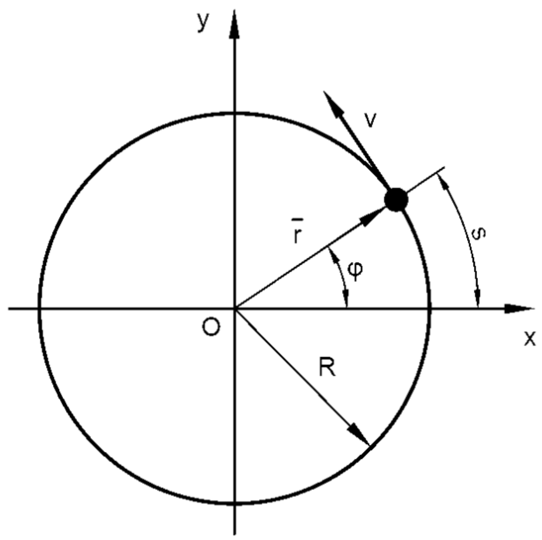


Figure 2.1.8

The angular position or angular displacement in radians

$$\varphi = \frac{s}{R}$$

The angular velocity is the rate of change of angular positions

$$\omega = \frac{\Delta\varphi}{\Delta t}$$

The position vector is referenced to the origin of the coordinate system.

$$\bar{r}(t) = R\left[\cos(\omega t)\bar{\imath} + \sin(\omega t)\bar{\jmath}\right]$$

The speed of a particle

$$v = \frac{\Delta s}{\Delta t} = R\frac{\Delta\varphi}{\Delta t} = R\omega$$

On the circular track the direction of the velocity vector is continually changing therefore:

$$\bar{a} = \frac{\Delta \bar{v}}{\Delta t}$$

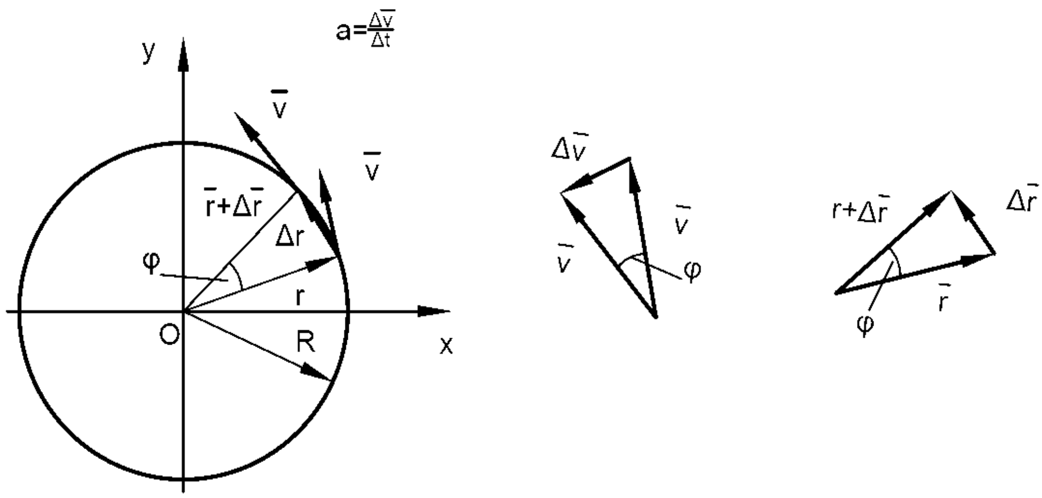


Figure 2.1.9

$$\frac{\Delta \bar{v}}{v} = \frac{|\Delta \bar{r}|}{\bar{r}} \frac{1}{\Delta t}$$

$$\frac{\Delta \bar{v}}{\Delta t} \frac{1}{v} = \frac{|\Delta \bar{r}|}{\Delta t} \frac{1}{|\bar{r}|}$$

$$|\bar{r}| = R$$

$$|\bar{v}| = v$$

$$\frac{a}{v} = \frac{v}{R};\ a_n = \frac{v^2}{R}$$

a_n, normal acceleration (toward the center).

When the object moves along a circular path with a non-constant speed, the second component of acceleration of a particle is called the tangential acceleration, it is the rate of change of angular velocity

$$\alpha = \frac{d\omega}{dt}$$

The acceleration is

$$a_t = \frac{dv}{dt} = \frac{d(R\omega)}{dt} = R\frac{d\omega}{dt} = R\alpha$$

The equations with constant angular velocity

$$\varphi = \varphi_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t;\ \alpha = constant$$

2.1.5 PLANAR PROJECTILE MOTION IN TERMS OF COMPONENT MOTION

Figure 2.1.10 shows the motion when a particle is launched with initial velocity in a vertical plane.

The curvilinear translational motion of the particle will be expressed now in terms of component motions in the x and y directions. In this model all air resistance effects will be neglected.

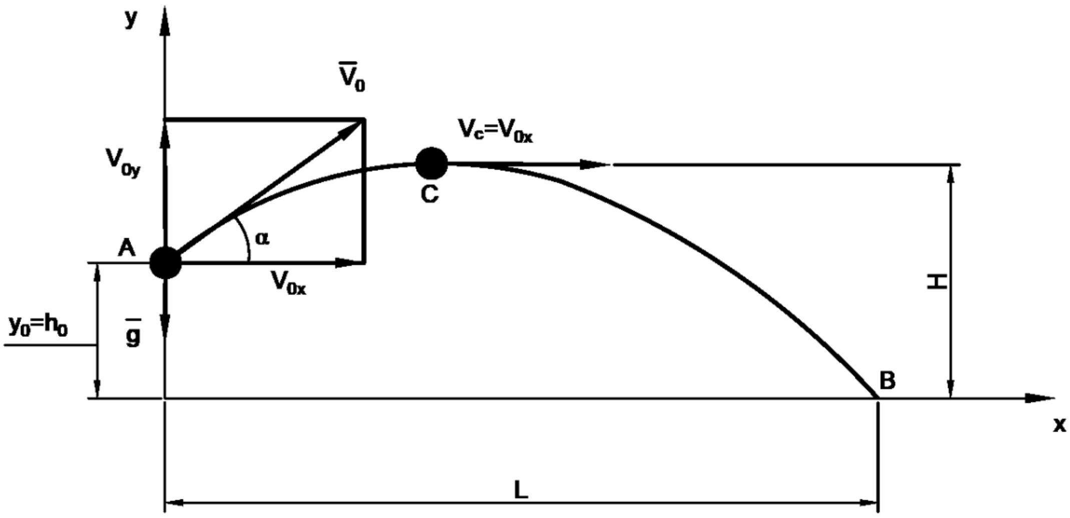


Figure 2.1.10

The motion of the particle in y direction is a case of free falling motion of body in gravitational field.

The magnitude of the acceleration a is equal to the magnitude g of the acceleration of gravitational filed.

$$a = -g$$

Equations for this case

$$v_y = v_{0y} - gt$$

$$y = y_0 + v_{0y}t - \frac{1}{2}\,gt^2$$

The gravitational acceleration in x direction is zero

Thus,

$$v_x\text{=}constant$$

Example Figure 2.1.10 shows a particle which is launched with initial velocity $v_0 = 80\ m/s$, $\alpha = 30°, h_0 = 100\ m$.

a. Find the maximum height reached by the particle
b. Find the duration of the flight and the distance L at which the particle strikes the ground.
c. Find the velocity with which the particle strikes on the ground

Initial conditions are

$$v_{0x} = v_0 \cos\alpha = 69.28\ ms^{-1}$$

$$v_{0y} = v_0 \sin\alpha = 40\ ms^{-1}$$

a. When the maximum height is reached

$$v_y = 0$$

$$0 = v_{0y} - gt$$

$$t = \frac{v_{0y}}{g} = \frac{40}{9.81} = 4.08\ s$$

The magnitude of the maximum heigh

$$y_{max} = h_0 + v_{0y}t - \frac{1}{2}\,gt^2 =$$

$$= 100 + 40 \cdot 4.08 - \frac{1}{2} \cdot 9.81 \cdot 4.08^2 = 181.6\ m$$

b. When the particle on strikes the ground

$$y = y_0 + v_{0y}t - \frac{1}{2}\,gt^2 =$$

$$= 100 + 40 \cdot t - 4.905 \cdot t^2 = 0$$

$$t = 10.16\ sec$$

$$L = v_{0x}t = 703.93$$

c. When the particle the ground t=10.16 sec

$$v_y = v_{0y} - gt = -59.67\ ms^{-1}$$

The magnitude of the striking velocity is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{69{,}28^2 + 59{,}57^2} = 91.43\ ms^{-1}$$

2.2 GENERAL PLANAR MOTION OF A RIGID BODY

The most general type of planar motion is the combination of rotation and translation. In the case of planar motion of a rigid body, all parts of the body move in parallel planes.

In the case of translational motion, every line in the body remains parallel to its original position at all time (Fig. 2.2.1).

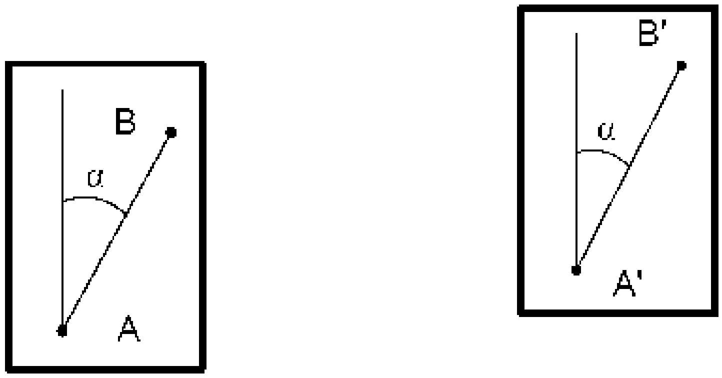


Figure 2.2.1

Fig. 2.2.2 shows the situation when the body moves with plane curvilinear motion

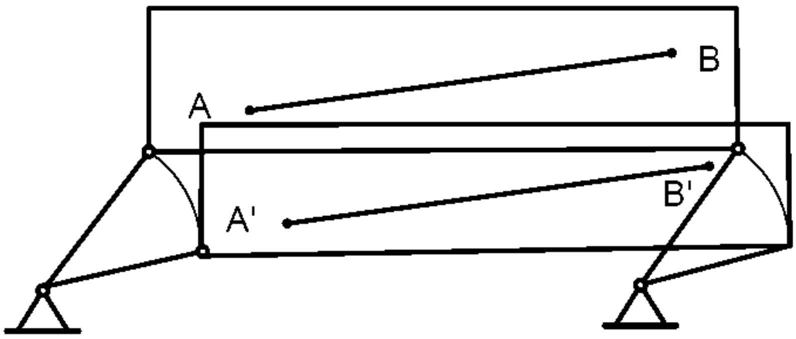


Figure 2.2.2

Any arbitrary line on the body is always parallel, in planar curvilinear paths. The velocity and acceleration at the two arbitrary A and B are

$$v_A = v_B$$
$$a_A = a_B$$

In the case of rotational motion, all particles move in circular paths about the axis of rotation. All lines in the body which are perpendicular to the axis of rotation rotate through the same angle at the same time. Circular motion of a point helps describe the rotating motion.

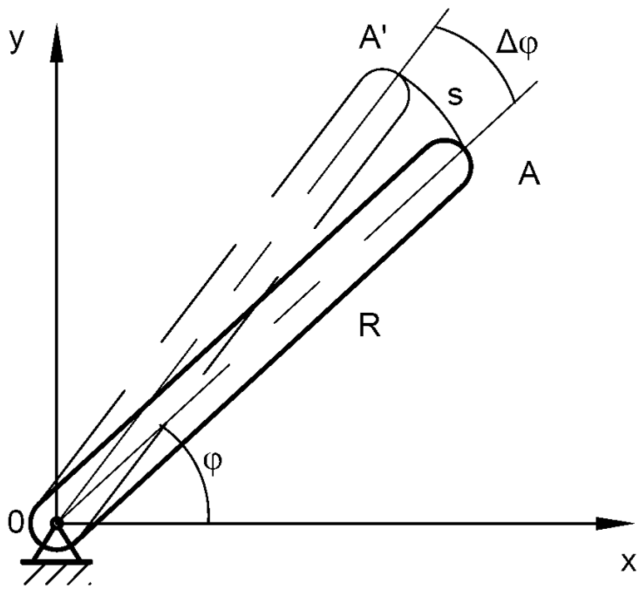


Figure 2.2.3

Fig. 2.2.3 shows a link of length R which rotates about fixed axis. The velocity of point A must be tangent to the path of motion. The magnitude of this velocity is given by

$$v = \frac{ds}{dt}$$

$$\frac{ds}{dt} = R\frac{d\varphi}{dt} = r\omega$$

2.2.1 ROTATION ABOUT FIXED AXIS

$$\overline{v} = \dot{\overline{r}} = \overline{\omega}\text{x}\overline{r} \text{ by vector differentiation}$$

$$|\overline{r}| = constant$$

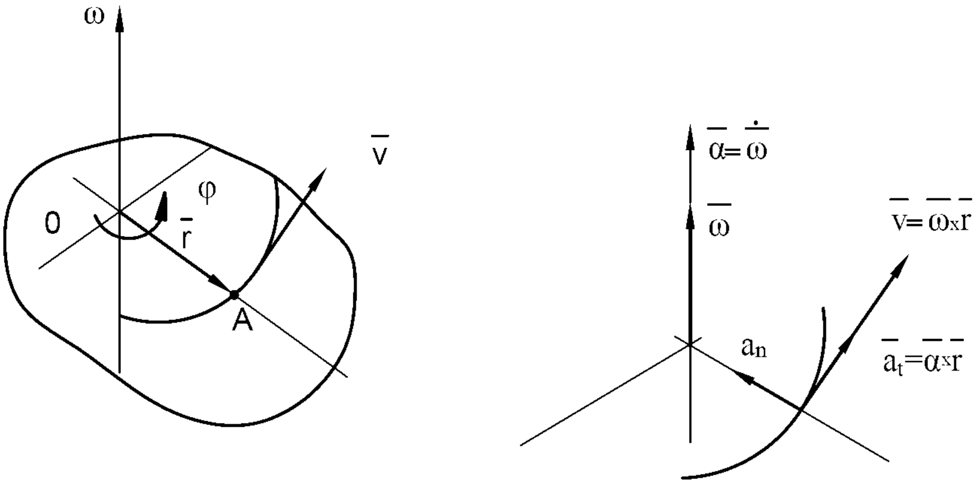


Figure 2.2.4

$$\overline{a} = \dot{\overline{v}} = \overline{\omega}\mathrm{x}(\overline{\omega}\mathrm{x}\overline{r}) + \overline{\alpha}\mathrm{x}\overline{r}$$

$$\overline{a_n} = \overline{\omega}\mathrm{x}(\overline{\omega}\mathrm{x}\overline{r}) = -\omega^2\overline{r}$$

$$\overline{a_t} = \overline{\alpha}\mathrm{x}\overline{r}$$

2.2.2 PLANAR KINEMATICS OF RIGID BODIES

The body in Fig. 2.2.5 is initially in position by specifying the $\overline{r_A}$ and $\overline{r_B}$. $\overline{AB}$ is a reference line in the rigid body.

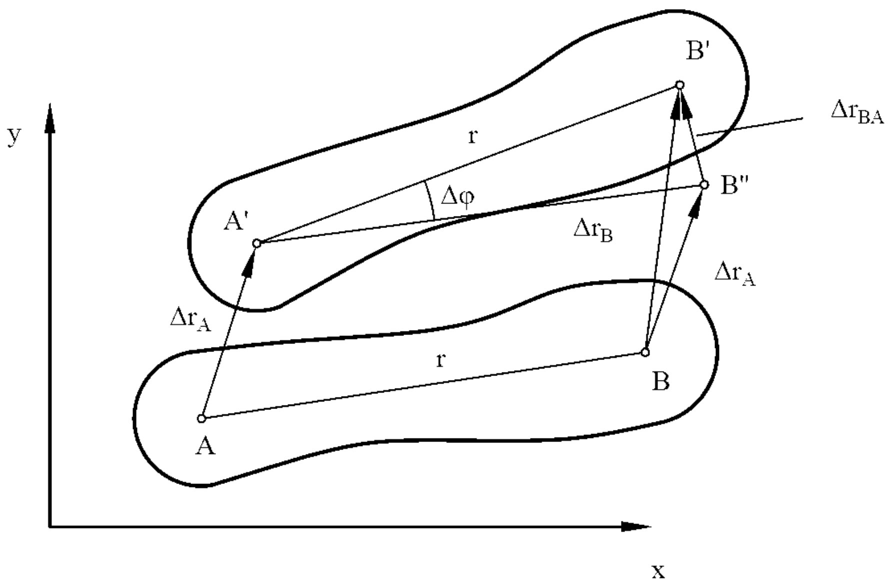


Figure 2.2.5

At a later time, the body is in position where the reference line is $\overline{A'B'}$.

$$\Delta r_B = \Delta r_A + \Delta r_{BA}$$

$$\Delta r_{BA} = r\Delta\varphi$$

$$v_{BA} = \lim_{\Delta t \to 0} \frac{\Delta r_{BA}}{\Delta t} = r\omega$$

Using $\overline{r}$ to represent vector $\overline{r_{AB}}$, relative velocity may be written as the vector

$$\overline{v_{BA}} = \overline{\omega} \mathrm{x} \overline{r}$$

Velocity of the B point is

$$\overline{v_B} = \overline{v_A} + \overline{\omega}\ \mathrm{x}\overline{r_{AB}}$$

2.2.3 INSTANT CENTER OF ROTATION

The translational velocities of the point A and B are known, it is desired to find the location of the instant center *(IC)* of rotation

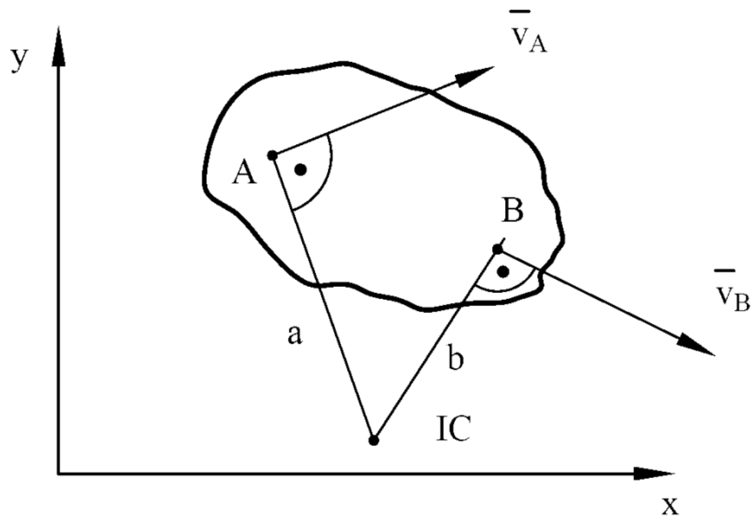


Figure 2.2.6

Line a is perpendicular to $\overline{v_A}$ and line b is perpendicular to $\overline{v_B}$. The point of intersection of these two lines is then the instant center of rotation of the body.

Example. The crank arm $\overline{OA}$ shown in Fig. 2.2.7 rotates counterclockwise at *10 rad/sec.* Find the velocity of the slider block and the angular velocity of link *AB*, for the position shown in the Fig 2.2.7.

$\overline{OA} = 400\ mm\ \overline{AB} = 700\ mm$

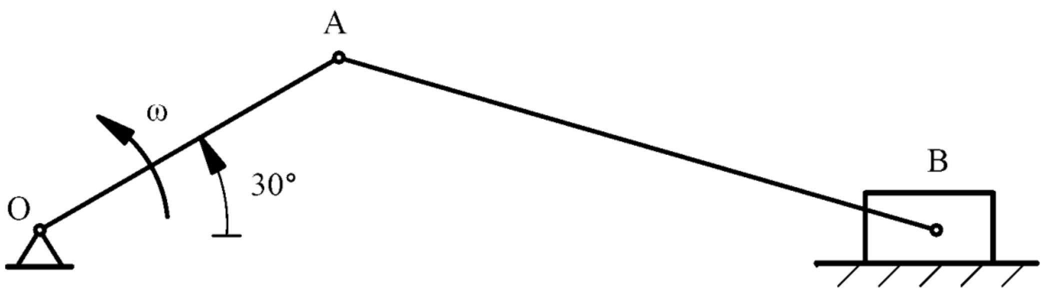
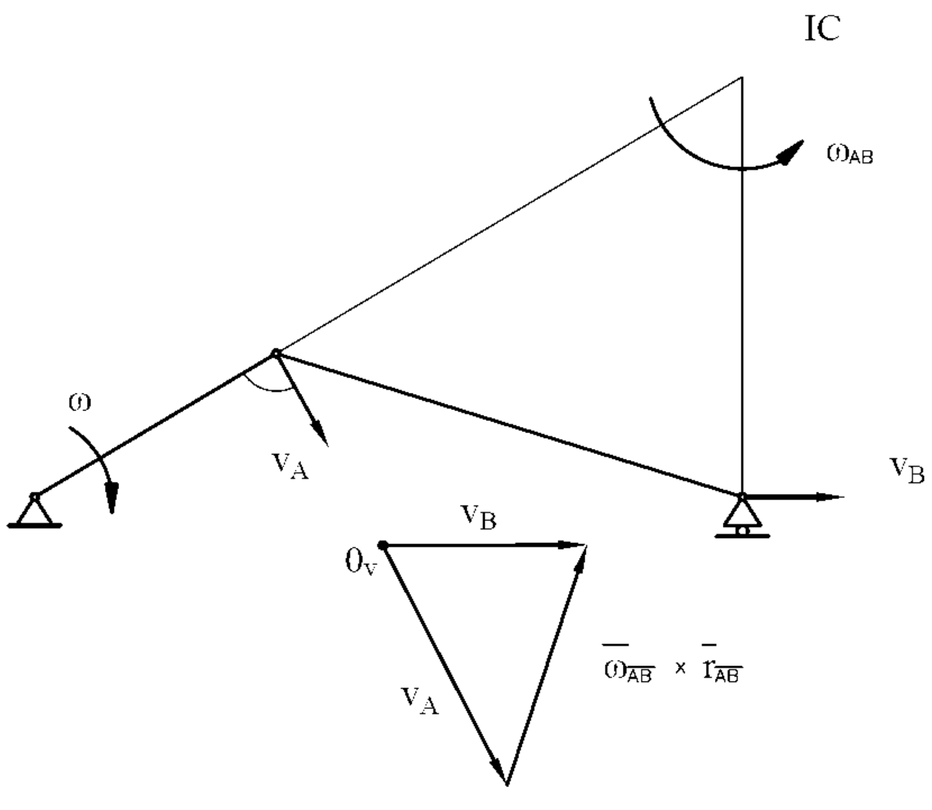


Figure 2.2.7

Solution.



$$\overline{r_{OA}} = 0.346\bar{i} + 0.2\bar{j}$$

$$\overline{v_A} = \overline{\omega}\,\mathrm{x}\overline{r_{OA}} = (-10\bar{k})\mathrm{x}(0.346\bar{i} + 0.2\bar{j}) = 2\bar{i} - 3.46\bar{j}$$

$$\overline{v_B} = \overline{v_A} + \overline{\omega_{AB}}\mathrm{x}\,\overline{r_{AB}} = 2\bar{i} - 3.46\bar{j} + \omega_{AB}\bar{k}\mathrm{x}\,(0.671\bar{i} - 0.2\bar{j})$$

$$v_B = 2 + 0.2\omega_{AB}$$

$$0 = -3.46 + 0.671\omega_{AB}$$

$$\omega_{AB} = 5.15\,\frac{1}{sec}\,;v_B = 3.03\,m/sec$$

2.2.4 STATE OF RIGID BODY ACCELERATION

Differentiating the velocity aquation results in the acceleration equation:

$$\overline{a_B} = \overline{a_A} + \overline{\alpha}\mathrm{x}\overline{r_{AB}} - \omega^2\overline{r_{AB}}$$

The motion of an arbitrary rigid body can be reduced to the superposition of a translation and a fixed axis rotation. Handling the translation of a rigid body is trivial, all points of the body move with the same velocity and acceleration, and we know how to deal with fixed axis rotations.

Fig. 2.2.8 shows *n*-t coordinates for general plane motion

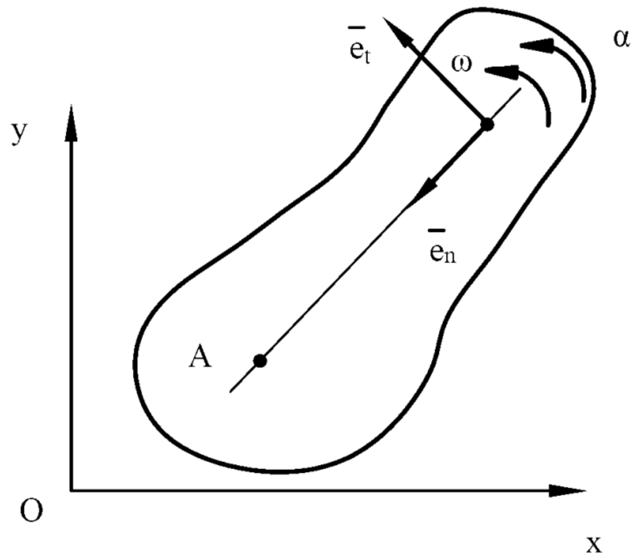


Figure 2.2.8

Fig 2.2.9 shows the decomposition of the absolute acceleration of point *B*.

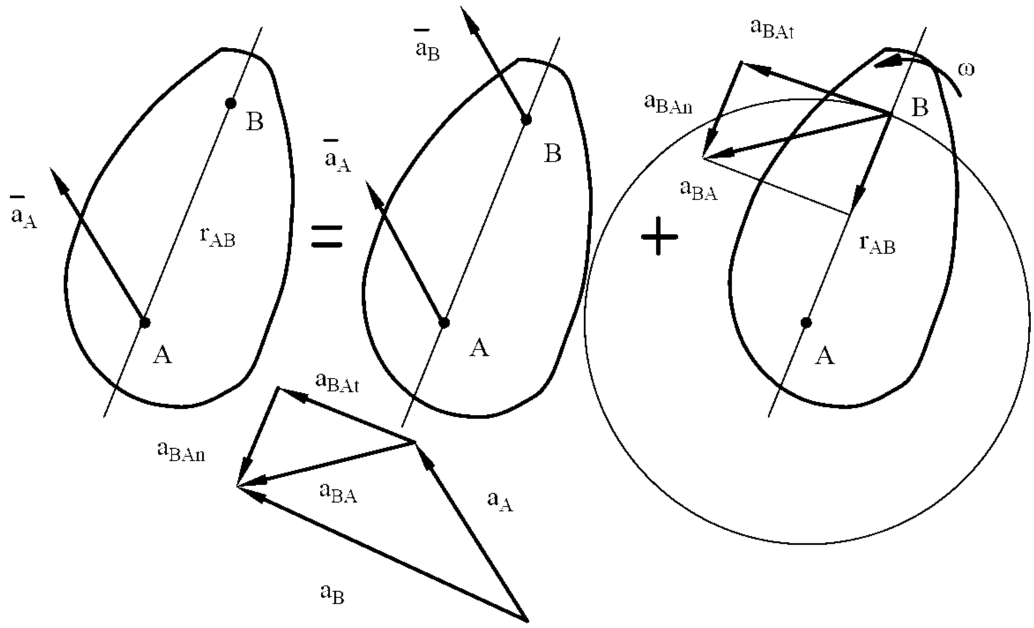


Figure 2.2.9

$$\overline{a_{BAn}} = r_{AB}\omega^2\,\overline{e_n}$$

$$\overline{a_{BAt}} = r_{AB}\alpha e_t$$

$$\overline{a_B} = \overline{a_A} + \alpha r_{AB}\overline{e_t} - \omega^2 r_{AB}\,\overline{e_n}$$

Example. Find the acceleration of the slider block, the position shown in Fig. 2.2.7.
$\omega = 10\ s^{-1};\ |\overline{r_A}| = 0.4\ m;\ \ |\overline{r_A}| = 0.7\ m.$

Solution.

$$\overline{a_A} = -\omega^2 r_{OA} = -34.6i - 20j\ ms^2$$

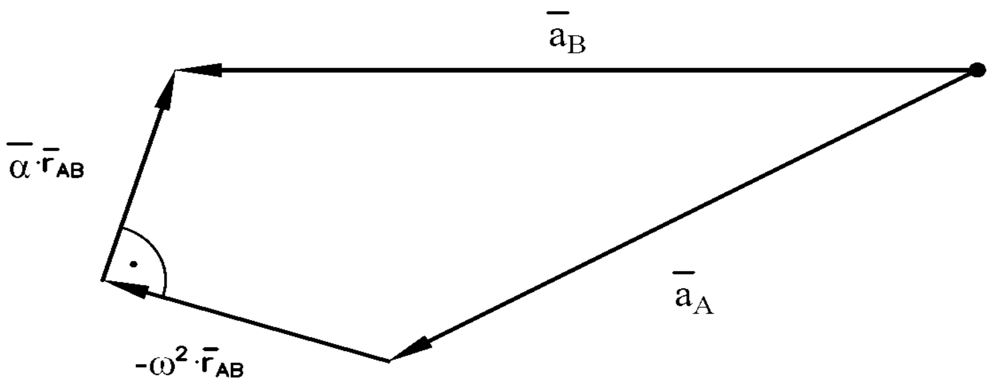
$$\overline{a_B} = \overline{a_A} + \overline{\alpha}\ \mathrm{x}\overline{r_{AB}} - \omega_{AB}^2\overline{r_{AB}}$$

$$a_B = -34.6 + 0.2\ \alpha - 17.8$$

$$0 = -20 + 0.671\alpha + 5.3$$

$$\alpha = 21.9\ \frac{1}{sec^2};\ \ a_B = -48\ ms^{-2}$$

Acceleration diagram



2.2.5 CENTER OF MASS OF RIGID BODY

The center of mass *(CM)* of a rigid body is a point in the body through which total weights act. It is also a point at which all mass of the body may be imagined to be concentrated. Center of mass are given by

$$\overline{r_c} = \frac{\int_m \overline{r}\, dm}{\int_m \, dm} = \frac{\int_v \; \overline{r}\, dV}{V}$$

If the material of body is homogeneous, the centroid of the volume of the body and the center of mass are coincident points. The center of mass (or gravity) of symmetric object is its geometric center if the density is uniform.

2.2.6 WEIGHT AND MASS

Weight is the gravitational force on an object.

Weight is a force parallel to the gravitational acceleration. On Earth, the gravitational acceleration is 9.81 ms^2.

$$W = mg$$

Density is the mass per unit volume

$$\varrho = \frac{m}{V}$$

2.2.7 CENTROID OF A COMPOSITE HOMOGENEOUS RIGID BODY.

In many problems the rigid body whose centroid is desired is not a simple shape (sphere, cylinder). In this case, it may be possible to subdivide the original body into elementary shapes whose centroidal coordinates known.

A centroidal coordinate of body has the general form

$$x_c = \frac{\int_v xdV}{\int_m V};\; y_c = \frac{\int_v ydV}{\int_m V}; z_c = \frac{\int_v zdV}{\int_m V}$$

Example

Find the x_c, y_c, z_c coordinates of the center of mass of the part shown in Fig. 2.2.10.

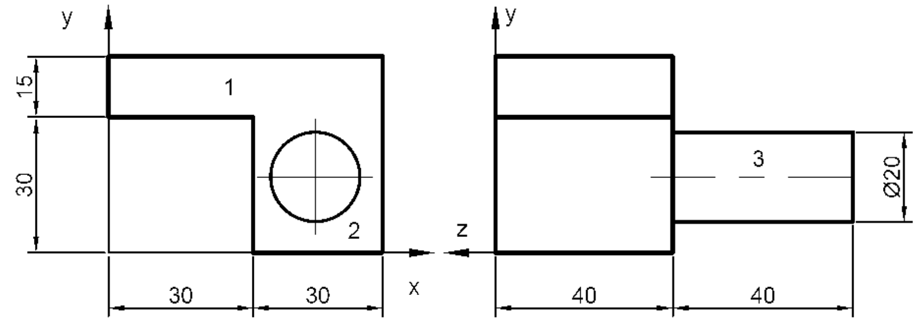


Figure 2.2.10

Solution.

$$V_1 = 1.5 \cdot 6.0 \cdot 4.0 = 36\, m^3$$

$$V_2 = 3 \cdot 3 \cdot 4 = 36\, m^3$$

$$V_3 = \frac{2^2\pi}{4} \cdot 4.0 = 12.56\, m^3$$

$$V = \sum V_i = 36\, cm^3$$

The coordinates of the three elementary volumes are

	x_c cm	y_c cm	z_c cm
1	3	3.75	-2
2	4.5	1.5	-2
3	4.5	1.5	-6

$$x_c = \frac{3 \cdot 36 + 4.5 \cdot 36 + 4.5 \cdot 12.56}{84.56} = 3.86\ cm$$

$$x_c = \frac{3.75 \cdot 36 + 1.5 \cdot 36 + 1.5 \cdot 12.56}{84.56} = 2.46\ cm$$

$$x_c = -\frac{2 \cdot 36 + 2 \cdot 36 + 6 \cdot 12.56}{84.56} = 2.596\ cm$$

2.2.8 MOMENTS OF INERTIA OF MASSES

Consider a body of mass m which is to be rotated about an axis ab Fig. 2.2.11. Dividing the body into elements of mass $\Delta m_{1,}\ \Delta m_2$ etc., we find that the resistance put up by the body is measure by the sum $r_1^2 \Delta m_1 + r_2^2 \Delta m_2 + \ldots$ This sum defines therefore the moment of inertia of the body with respect to the axis *ab.*

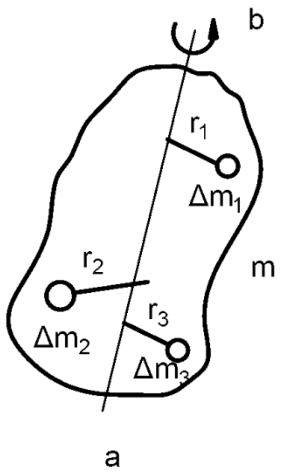


Figure 2.2.11

Increasing the number of elements, we find that moment of inertia is equal, at limit, to the integral

$$I = \int r^2\, dm$$

2.2.9 PARALLEL – AXIS THEOREM

Denoting by d the distance between an arbitrary axis $a'b'$ and a a parallel centroidal axis to ab as shown on Fig. 2.2.12. we may write the following general relation between the moment of inertia I_{ab} of the body with respect to $a'b'$ and its moment of inertia $I_{a'b'}$ with respect to $a'b'$:

$$I_{a'b'} = I_{ab} + md^2$$

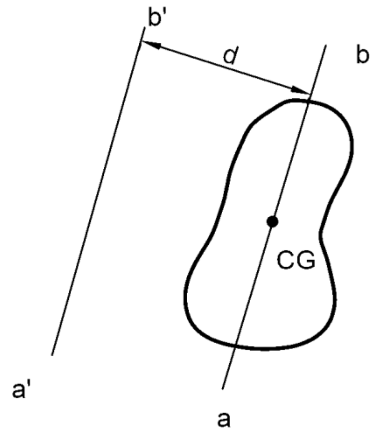


Figure 2.2.12

Mass moments of inertia of common geometric shapes are shown in Fig. 2.2.13.

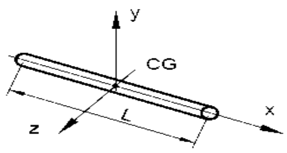
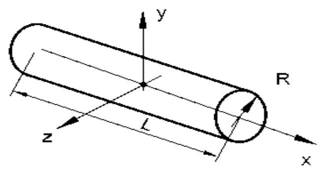
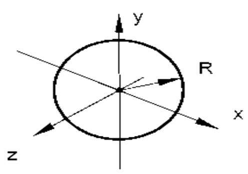
Slender rod		$I_z = I_y = \frac{m \cdot L^2}{12}$
Circular cylinder		$I_x = \frac{m \cdot R^2}{12}$ $I_y = I_z = \frac{m}{12}(3R^2 + L^2)$
Sphere		$I_x = I_y = I_z =$ $= \frac{2}{5} mR^2$

Figure 2.2.13

2.2.10 MOMENTS OF INERTIA OF COMPOSITE BODIES

The moments of inertia with respect to a given axis of a body made of simple shapes may be obtained by the sum of the moments of inertia of the component parts about the desired axis..

Example. Determine the mass moment of inertia of the steel link shown in Fig. 2.2.14 (density of steel 7850 kg/m^3)

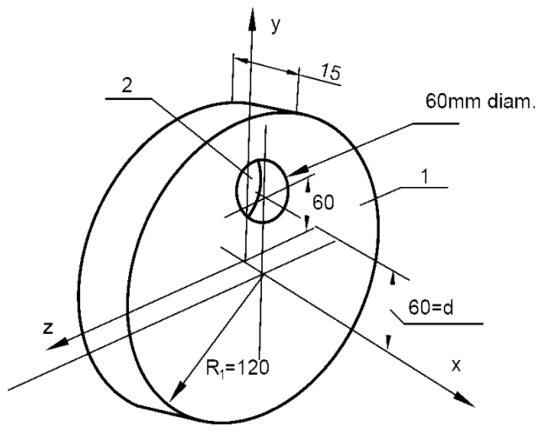


Figure 2.2.14

Solution.

$$I_x = \frac{1}{2}m_1R_1^2 - \left(\frac{1}{2}m_1R_1^2 + m_2d^2\right) = 0.039\ kgm^2$$

2.3 MECHANISMS

Kinematic analysis of structures is of great importance in mechanical engineering. Mechanisms can be divided into planar mechanisms and spatial mechanisms. In planar mechanisms, all of the relative motions of the rigid bodies are in one plane or in parallel planes. The mechanisms examined now have a single degree of freedom.

2.3.1 SCOTCH-YOKE MECHANISMS

This mechanism is used for converting rotary motion into a linear motion Fig. 2.3.1

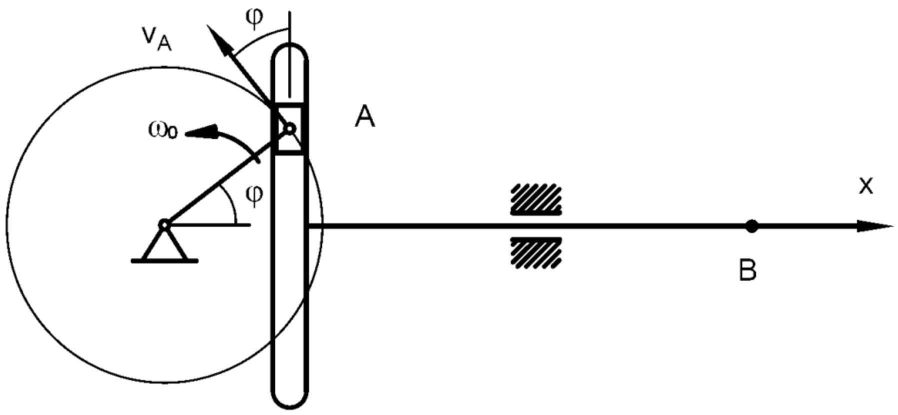


Figure 2.3.1

The crank rotates at constant angular speed ω_0, defining the speed and acceleration of the slider.

The speed:

$$v_B = v_A sin\varphi = r\omega_0 sin\omega_0 t$$

The acceleration:

$$a_B = \frac{dv_B}{dt} = r\omega_0^2 cos\omega_0 t$$

The displacement:

$$x_B = \int_0^t v_B(t)dt = r\omega_0 \int_0^t sin\omega_0 tdt = r(1 - \cos\omega_0 t)$$

2.3.2 SLIDER CRANK MECHANISMS

This mechanism is composed of three important parts: the crank which is rotating, the slider and the connecting rod which joins the parts together.

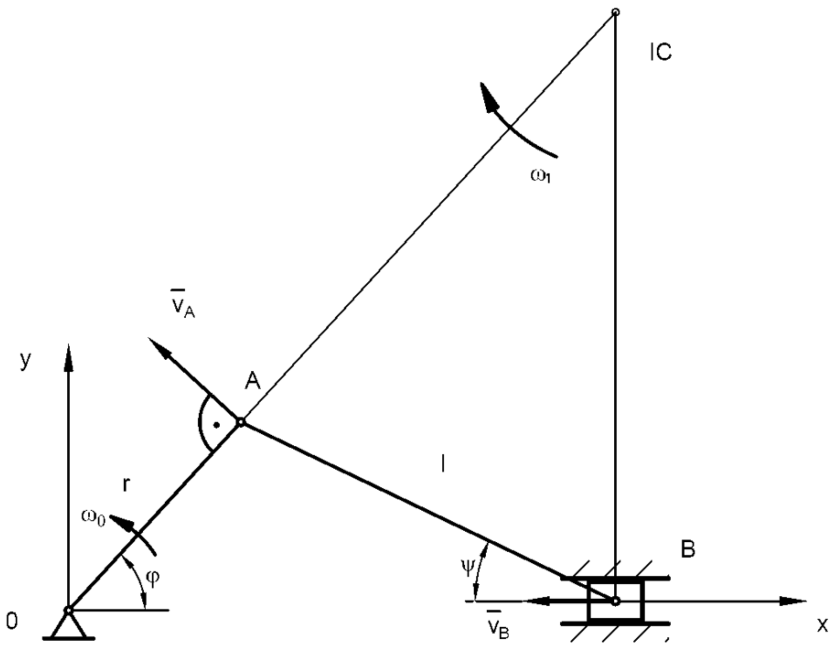


Figure 2.3.2

The crank arm r shown in Fig. 2.3.2 rotates counterclockwise at ω_0. Find the velocity and acceleration of the slider.

The speed:

$$|v_A| = r\omega_0;\ \overline{r_{AB}} = lcos\psi\,\overline{i} - lsin\psi\,\overline{j}$$

$$\overline{v_A} = r\omega_0(\,-sin\varphi\,\overline{i} + cos\varphi\,\overline{j})$$

$$\overline{v_B} = \overline{v_A} + \overline{\omega_1}\mathrm{x}\overline{r_{AB}}$$

$$\overline{\omega_1}\mathrm{x}\overline{r_{AB}} = -\,l\omega_1 sin\psi\,\overline{i} - l\omega_1 cos\psi\,\overline{j}$$

$$v_B = -v_A sin\varphi + l\omega_2 sin\psi$$

$$0 = v_A cos\varphi - l\omega_2 cos\psi$$

$$\omega_2 = -\frac{v_A cos\varphi}{l cos\psi}$$

$$v_B = -v_A sin\varphi + v_A cos\varphi tg\psi$$

$$tg\Psi = \frac{\frac{r}{l} sin\varphi}{\sqrt{1 - \frac{r^2}{l^2} sin^2\varphi}} \approx \frac{r}{l}\ sin\varphi$$

if:

$$\frac{r}{l} \leq \frac{1}{4}$$

$$v_B = r\omega_0(\ -sin\omega_0 t + \frac{r}{2l} sin2\omega_0 t)$$

$$a_B = r\omega_0^2(\ -cos\omega_0 t + \frac{r}{l} cos2\omega_0 t)$$

The displacement:

$$x_B = \int_0^t v_B dt = r\left[(1 - cos\omega_0 t) + \frac{r}{4l}(1 - cos2\omega_0 t)\right]$$

2.3.3 FOUR-BAR LINKAGE MECHANISM

The four-bar linkage mechanism show in Fig. 2.3.3

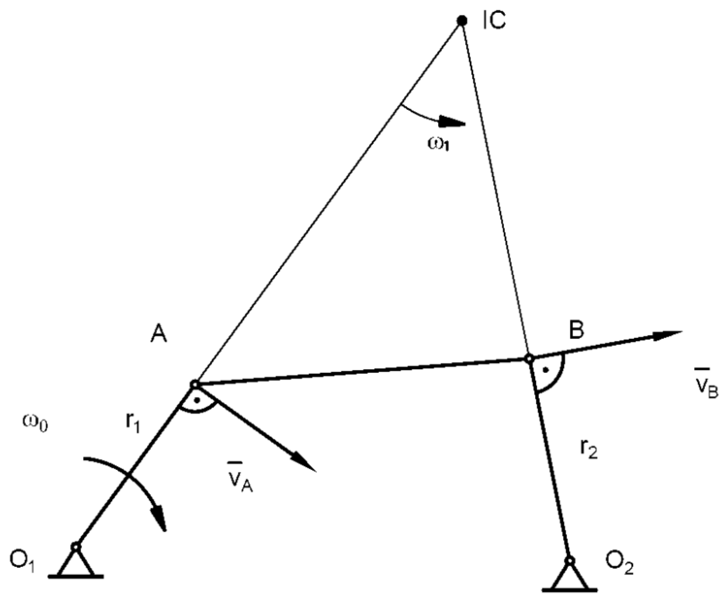
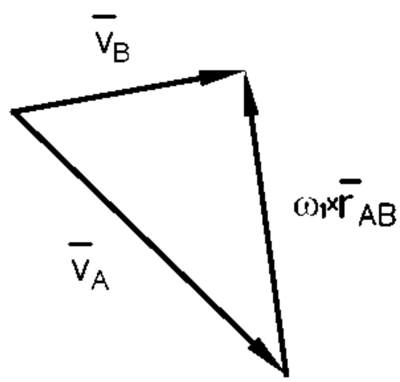


Figure 2.3.3

The mechanism has two rotating links which have fixed pivots. One of the levers would be input rotation, while the other would be output rotation. The two levers are connected by the coupler link *l*.

The crank rotates clockwise. Find the velocity of point *B*. Solve the problem graphically Fig. 2.3.4

$$v_A = r_1 \omega_0$$



Velocity diagram

Figure 2.3.4

2.4 KINETICS OF PARTICLES

The motion of particles is governed by Newton's three laws of motion.

First law: A particle originally at rest, or moving in a straight line at constant velocity, will remain in this state if the resultant force acting on the particle is zero.

Second law: If the resultant force on the particle is not zero, the particle experiences an acceleration in the same direction as the resultant force. This acceleration has a magnitude proportional to the resultant force.

Third law: Mutual forces of action and reaction between two particles are equal, opposite, and collinear.

The first and third laws were used in developing the concepts of statics. Newton's second law forms the basis of study of dynamics.

Mathematically, Newton's second law of motion can be written:

$$\overline{F} = m\overline{a}$$

where $\overline{F}$ is the resultant unbalanced force acting on the particle, and $\overline{a}$ is the acceleration of the particle. The positive scalar m is called the mass of the particle.

2.4.1 GRAVITATIONAL FORCE

The studies and observations of planetary motions led Newton to the conclusion to the law of universal gravitation. Two particles of mass M and m attract each other with equal and opposite force along the line connecting the particles.

$$F = G\,\frac{Mm}{r^2}$$

where *G:* constant of gravitation.

$\mathrm{G} = 66.73 \cdot 10^{-12}\, Nm^{-2}\, kg^{-2}$

$\mathrm{M} = 5.9736 \cdot 10^{24}\, kg;$

$r = 6372.797\, km$

For particle of mass m on the Earth's surface.

$$W = m\,\frac{MG}{R^2} = mg$$

$$g = 9.81\, m/s^2$$

2.4.2 IMPULSE-MOMENTUM THEOREM

Linear momentum I is

$$\bar{I} = m\,\bar{v}$$

The momentum is directly proportional to the object's mass *(m)* and velocity *(v)*. Momentum is a vector and has the same direction as velocity *($\bar{v}$).*

Newton's second law of motion in terms of momentum:

$$\bar{F} = \mathrm{m}\,\frac{\Delta\bar{I}}{\Delta t}$$

thus

$$\bar{F}\Delta t = \Delta\bar{I} = \mathrm{m}\overline{v_2} - \mathrm{m}\overline{v_1}$$

is known as impulse or momentum.

The effect of a force on an object depends on how long it acts, as well as the magnitude of the force.

Example: The system of two blocks in Fig. 2.4.1 is released from rest at $t = 0$.

Find the acceleration of the two blocks and the cable tensile force. The pulley is massless, and the pulley bearings are frictionless.

$$m_1 = 40\ kg;\ m_2 = 60\ kg$$

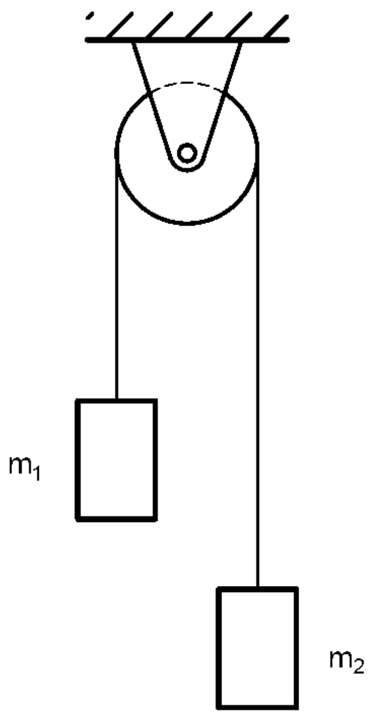


Figure 2.4.1

Solution. From physical consederation, it may be concluded that block m_1, moves upward and block m_2, moves download.

$$g = 9.81\ m/s^2$$

The equations of motion are

$$\text{Block } m_1 : S - m_1 g = m_1 a$$

$$\text{Block } m_2 : g - s = m_2 a$$

$$a = \frac{m_2 g - m_1 g}{m_1 + m_2} = 1.96\ \mathrm{m}/s^2$$

$$S = m_1(g + a) = 470.8\ N$$

Example. A *60 kg* crash test dummy is travelling in a car moving at 30 m/s. The car collides with a wall and comes to rest in 0,1 sec. How much force does the dummy experience in the collision?

Solution. First, we calculate the change in momentum

$$\Delta I = \mathrm{m}\Delta v = 60(0 - 30) = -1800\ kg\ m/s$$

Now applying the impulse-momentum theorem:

$$Ft = \Delta I$$

$$F = -\frac{1800}{0.1} = -18000\ N$$

This massive amount of force would cause deadly injuries to a real passenger.

2.4.3 ANGULAR MOMENTUM OF A PARTICLE

The angular momentum $\overline{\Pi}$ is defined as the cross-product of $\overline{r}$ and $\overline{I}$, and is perpendicular to the plane contain $\overline{r}$ and $\overline{I}$

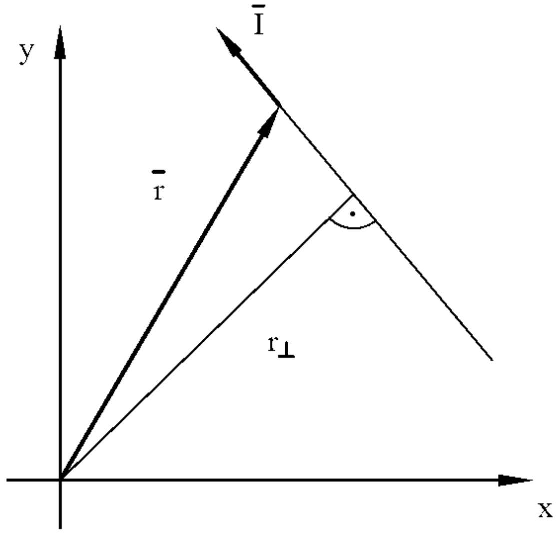


Figure 2.4.2

$$\overline{\Pi} = \overline{r}x\overline{I} = \overline{r}xm\overline{v}$$

The magnitude of the angular momentum:

$$\Pi = r_\perp mv$$

Relationship between net torque and angular momentum on the particle.

$$\frac{d\overline{\Pi}}{dt} = \frac{d(\overline{r}\mathrm{x}\overline{I})}{dt} = \frac{d\,\overline{r}}{dt}x\overline{I} + \frac{d\overline{I}}{dt}x\overline{r}$$

$$\frac{d\overline{r}}{dt}x\overline{I} = \overline{v}\mathrm{x}m\overline{v} = m(\overline{v}\mathrm{x}\overline{v}) = \overline{0}$$

$$\frac{d\overline{\Pi}}{dr}\mathrm{x}r = \overline{0} + \frac{d\overline{I}}{dt}\mathrm{x}r = \overline{F}\mathrm{x}\overline{r}$$

$$\frac{d\overline{\Pi}}{dt} = \overline{M}$$

2.4.4 WORK-ENERGY METHODS FOR PARTICLES

Work is equal to displacement multiplied by the component of force along the displacement Fig. 2.4.3

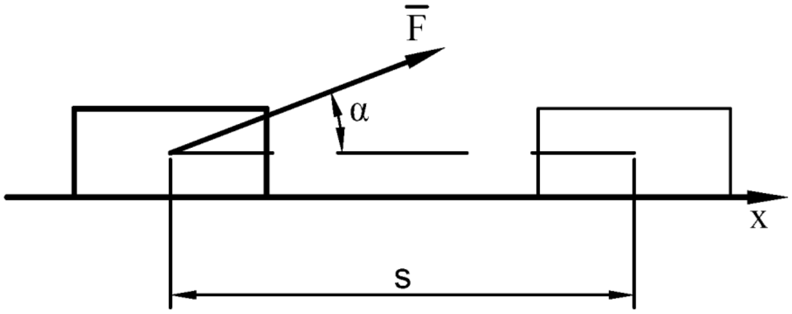


Figure 2.4.3

The component of F along s is $Fcos\alpha$, the work is:

$$W = Fscos\alpha$$

In general form the work:

$$W = \overline{F}\,\overline{s}$$

Work is scalar quantity, and units of work Nm.

Kinetic energy of an object is a measure of its ability to do work. The translational kinetic energy of an object of mass m and velocity v is:

$$T = \frac{1}{2}mv^2$$

Figure 2.4.4 shows a particle which moves along a curved pathdenoted by *s.* The resultant force $\overline{F}$ acts on the particle, with the normal and tangential components F_n and F_t.

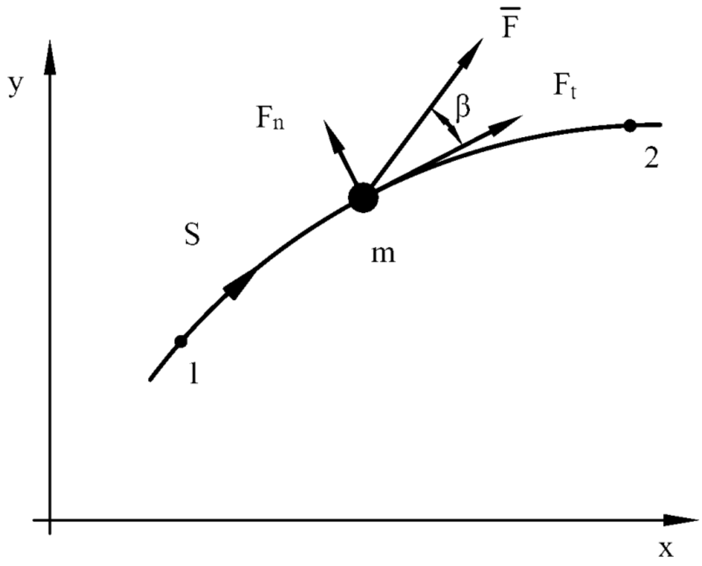


Figure 2.4.4

Newton's second law in the tangential direction is written as

$$F_t = ma_t$$

The tangential acceleration a_t may be written as

$$a_t = \frac{dv}{dt}$$

where v is the scalar magniture of the velocity.

The force F_t may be written as

$$F_t = Fcos\beta$$

$$Fcos\beta = m\frac{dv}{dt}$$

The term v is defined in terms of the length coordinate s

$$v = \frac{ds}{dt}$$

from which

$$dt = \frac{ds}{v}$$

$$Fcos\beta = m\frac{dv}{ds/v} = mv\frac{dv}{ds}$$

$$(F cos\beta)ds = mvdv$$

Equation is now integrated between two positions *1* and *2* shown in the figure, and the result is

$$\int_1^2 (F cos\beta)ds = \int_1^2 mvdv = m\int_1^2 vdv = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

The work done by F is equal to the change in kinetic energy of the object

$$W = T_2 - T_1$$

This relationship is called work-energy theorem.

2.4.5 COLLISION OF PARTICLES

Figure 2.4.5 shows two masses moving along the same axis in opposite direction. The masses have the constant velocities v_1 and v_2. Figure 2.4.5 shows the free-body diagram during the collision, or impact.

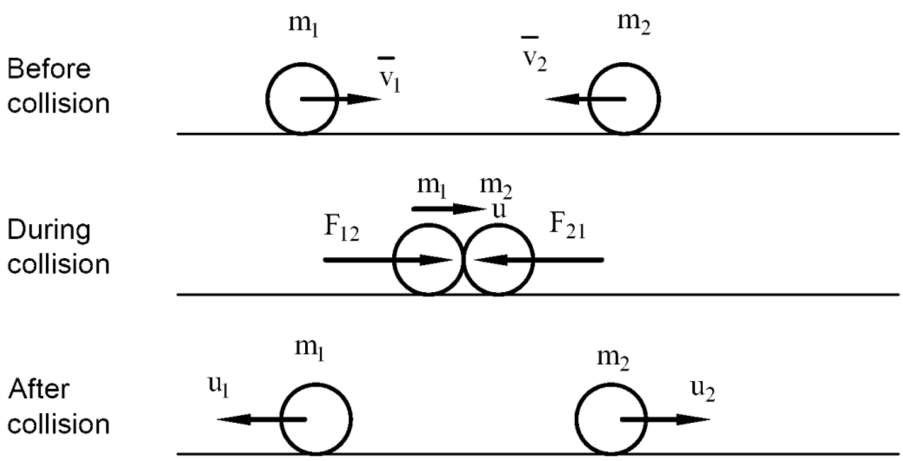


Figure 2.4.5

During this period of contact a compressive force acts between the two masses.

$$\overline{F_{12}} + \overline{F_{21}} = \overline{0}$$

At the moment of the collision the two masses move together with speed *u.*

The equation of impulse-momentum for each mass are

Mass m_1

$$-\int \overline{F_{21}}dt = m_1\overline{u} - m_1\overline{v_1}$$

Mass m_2

$$\int \overline{F_{12}}dt = m_2\overline{u} - m_2\overline{v_2}$$

These equations are added

$$m_1\overline{v_1} + m_2\overline{v_2} = (m_1 + m_2)\overline{u}$$

$$\overline{u} = \frac{m_1\overline{v_1} + m_2\overline{v_2}}{m_1 + m_2}$$

When two physical bodies collide, there is a permanent energy loss as a result of effects occurring in the material during the deformation and restitution phases of the impact process. A Term which is a measure of the energy loss during impact will be introduced now. This quantity is referred to as the coefficient of restitution, designated by the symbol e is given by

$$0 \leq e \leq 1$$

In the case of elastic impact $e = 1$, and if an impact is non-elastic, it is referred to as inelastic, $e = 0$.

The impulse in the second period

$$\int_{t_1}^{t_2} \overline{F}dt = e\int_{0}^{t_1} \overline{F}dt$$

The equations of impulse momentum for each mass are

Mass m_1

$$m_1\overline{u_1} - m_1\overline{u} = e(m_1\overline{u} - m_1\overline{v_1})$$

Mass m_2

$$m_2\overline{u_2} - m_2\overline{u} = e(m_2\overline{u} - m_2\overline{v_2})$$

$$\overline{u_1} = (1 + e)\overline{u} - e\overline{v_1}$$

$$\overline{u_2} = (1+e)\overline{u} - e\overline{v_2}$$

$$e = -\frac{u_2 - u_1}{v_2 - v_1} = -\frac{relative\ velocity\ after\ impact}{relative\ velocity\ before\ impact}$$

Example. Two spheres approach each other with constant velocity along rectilinear paths, as shown in Fig 2.4.6. The coefficient of restitution is assumed to be 0.85.

Find the magnitudes and directions of the velocities of the two spheres after the impact

$$m_1 = 1.5\ kg; m_2 = 2\ kg$$

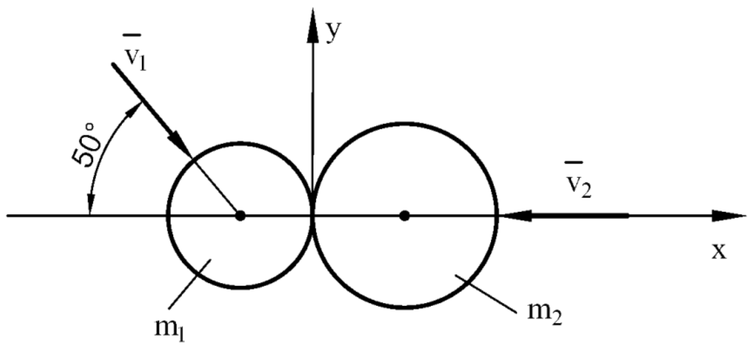


Figure 2.4.6

Solution.

$$v_{1x} = 2.8cos50° = -1.8\ m/s$$

$$v_{1y} = 2.8sin50° = 2.14\ m/s$$

The common velocity in collision

$$u = -\frac{1.5 \cdot 1.8 - 2 \cdot 4.,5}{1.5 + 2} = -1.8\ m/s$$

velocities after impact

$$u_{1x} = (1 + 0.85)(-1.8) - 0.85 \cdot 1.8 = -4.86\ m/s$$

$$\overline{u_1} = -4.86\overline{i} - 2.14\overline{j}\ m/s$$

$$u_2 = (1 + 0.85)(-1.8) + 0.85 \cdot 4{,}5 = 0.495\ m/s$$

The magnitudes of the two velocities after impact are:

$$u_1 = \sqrt{4.86^2 + (2.8sin50°)^2} = 5.31\ m/s$$

$$u_2 = 0.495\ m/s$$

Fig. 2.4.7 shows the graphical solution of particle collision

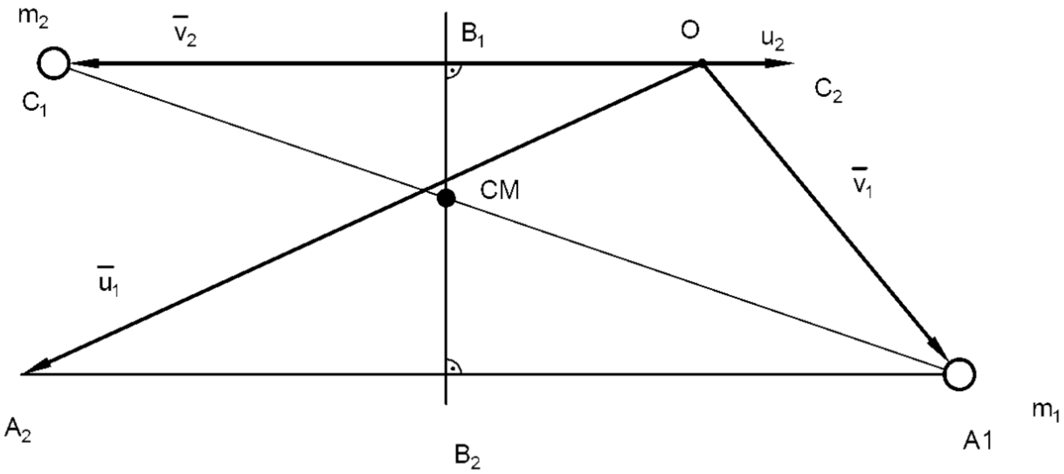


Figure 2.4.7

$$\overline{B_1C_2} = e\overline{B_1C_1}$$

$$\overline{B_2A_2} = e\overline{B_2A_1}$$

2.5 KINETICS OF RIGID BODIES

The complete description of the general plane motion of rigid bodies is given by two equations

$$\overline{F} = m\overline{a_c}$$

$$M_0 = I_0\alpha$$

The first equation describes the translational motion of the CM, while the second describes the rotational motion about the CM.

The second equation is written in scalar form.

2.5.1 KINETIC ENERGY OF A RIGID BODY IN PLANAR MOTION

Fig. 2.5.1 shows a rigid body which is in planar rotational motion about a fixed point.

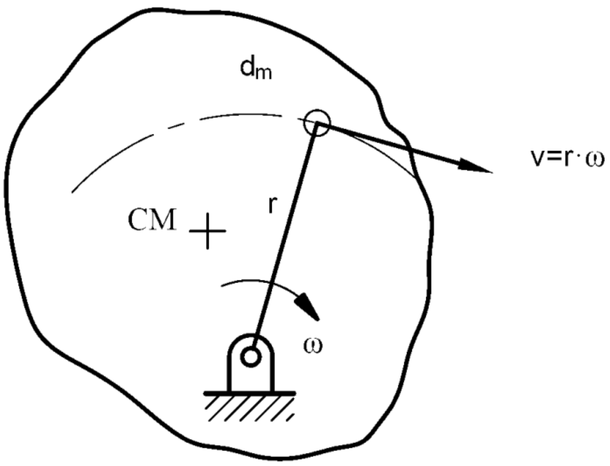


Figure 2.5.1

The absolute velocity of the typical mass element *dm* is

$$v = r\omega$$

The kinetic energy of this element is

$$dT = \frac{1}{2}(dm)v^2 = \frac{1}{2}dm(r\omega)^2$$

The total kinetic energy of the body is found by summing of all the individual elements

$$T = \int_{vol} dT = \frac{1}{2}\omega^2 \int_{vol} r^2 dm$$

$$I = \int_{vol} r^2 dm$$

The kinetic energy T has the final form

$$T = \frac{1}{2} I\omega^2$$

For the present problem, I is the mass moment of inertia of the body about the fixed point which is the instant center. Using the transfer theorem for moments of inertia, we get

$$I = I_0 + mr_0^2$$

where I_0 is the mass moment of inertia of the body about the CM.

The kinetic energy term now has the form of

$$T = \frac{1}{2}(I_0 + mr_0^2)\omega^2 = \frac{1}{2} I_0\omega^2 + \frac{1}{2} mr_0^2\omega^2$$

Since $v_0 = r_0\omega$, the final form

$$T = \frac{1}{2} mv_0^2 + \frac{1}{2} I\omega_0^2$$

Example. The rod of weight 60 N in Fig. 2.5.2 is initially supported by a cable.

a. Find the angular acceleration of the rod at the instant that the string is cut.
b. Find the angular velocity of the rod at this instant.
c. Determine the reaction force at point A.

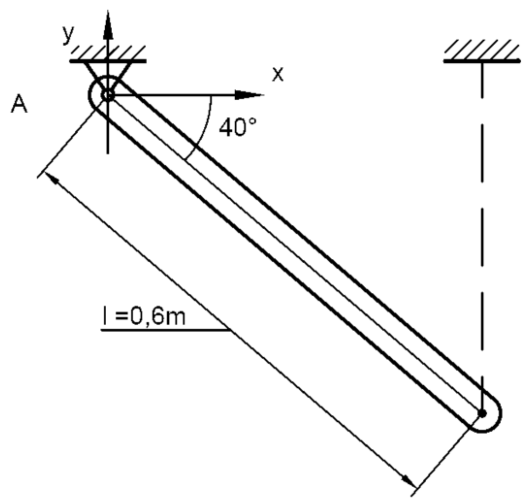
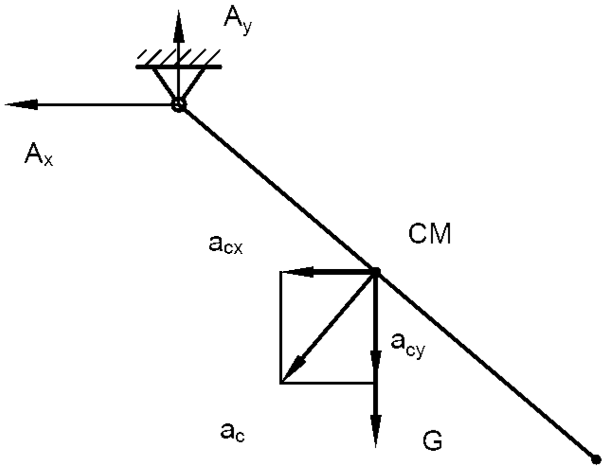


Figure 2.5.2

Solution.



$$M_A = 0.3cos40°60 = 13.8\ Nm$$

$$I_a = \frac{ml^2}{3} = \frac{60 \cdot 0.6^2}{9.81 \cdot 3} = 0.734\ kgm^2$$

$$\alpha = \frac{M_A}{I_a} = \frac{13.8}{0.734} = 18.8\ 1/s^2$$

$$a_c = 0.3 \cdot 18.8 = 5.64\ m/s^2$$

$$\overline{a_c} = -3.625\bar{i} - 4.32\bar{j}\ m/s^2$$

$$\sum F_{yi} = Y_A - G = -a_{cy} - ma_{cy}$$

$$Y_A = G - ma_{cy} = 33.58\ N$$

$$X_A = ma_{cx} = -22.17\ N$$

2.5.2 IMPULSE-MOMENTUM EQUATION FOR RIGID BODIES

Newton's second law for rotation of the body about the fixed point is:

$$M = I\alpha = I\frac{d\omega}{dt}$$

Equation $M = I\alpha$ integrated over a time interval with end points *1* and *2*. The result is

$$\int_1^2 Mdt = \int_1^2 Id\omega = I(\omega_2 - \omega_1)$$

The quantity $\int_1^2 Mdt$ is the impulse of the moment M.

Example. The disk in Fig. 2.5.3 rotates with angular velocity $\omega = 10\ s^{-1}$.

How long will it take the disc to stop if we break?

$$F_N = 100\ N;\ \mu = 0.2; d = 400\ mm; m = 80kg$$

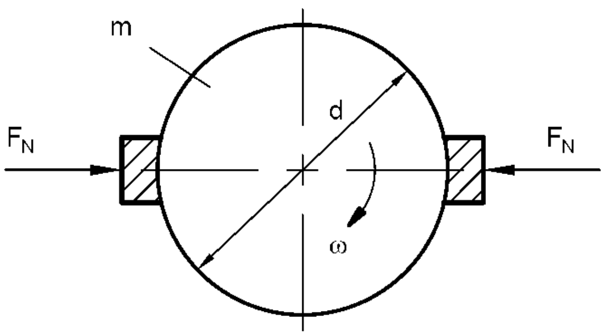


Figure 2.5.3

Solution.

Moment of inertia

$$I = \frac{1}{2}mr^2 = \frac{1}{2}\cdot 80\cdot 0.2^2 = 1.6\ kgm^2$$

breaking torque:

$$M = -d\cdot\mu\cdot F_N = -0.4\cdot 0.2\cdot 100 = -8\ Nm$$

$$M\Delta t = I(\omega_2 - \omega_1)$$

$$\Delta t = \frac{1.6(0-10)}{-8} = 2\ sec$$

Example. A 5 kg spherical ball with radius of 0.05 m is placed on a ramp as shown in Fig. 2.5.4. If the ball rolls without slipping, what is the velocity of the ball at the bottom of the ramp?

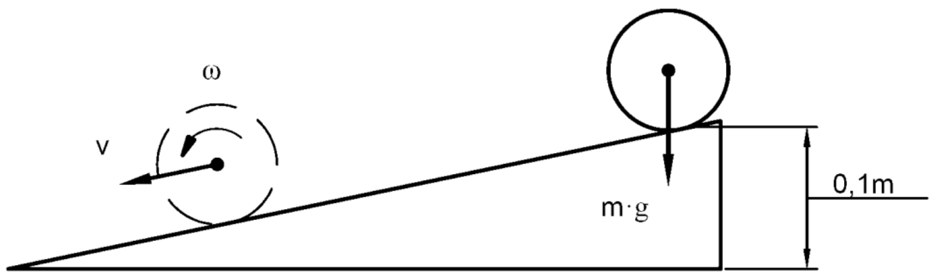


Figure 2.5.4

Solution.

$$W = T_2 - T_1$$

$$W = mgh$$

$$T_2 = \frac{1}{2}mv^2 + \frac{1}{2}\,\mathrm{I}\,\omega^2; \quad T_1 = 0$$

$$T_2 = \frac{1}{2}(m + \frac{2}{5}\,mr^2\frac{1}{r^2})v^2$$

$$mgh = \frac{1}{2}m(1 + \frac{2}{5})v^2$$

$$v = \sqrt{\frac{9.81 \cdot 0.1}{0.7}} = 1.184\,m/s$$

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Machine Component Design and Mechanics

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